Mathematics

Fairfield Public Schools

Introduction to Calculus 50

INTRODUCTION TO CALCULUS 50

Critical Areas of Focus

Introduction to Calculus 50 course is designed for the student who has completed Pre-Calculus and wishes to be introduced to a college calculus experience. To be successful, students must be motivated learners who have mathematical intuition, a solid background in the topics studied in previous courses and the persistence to grapple with complex problems. The critical areas of focus for this course will be in three areas: (a) functions, graphs and limits, (b) differential calculus (the derivative and its applications), and (c) integral calculus (anti-derivatives and their applications).

1) Students will build upon their understanding of functions from prior mathematics courses to determine continuity and the existence of limits of a function both graphically and by the formal definitions of continuity and limits. They will use the understanding of limits and continuity to analyze the behavior of functions as they approach a discontinuity or as the function approaches $\pm\infty$.

2) Students will analyze the formal definition of a derivate and the conditions upon which a derivative exists. They will interpret the derivative as the slope of a tangent line and the instantaneous rate of change of the function at a specific value. Students will distinguish between a tangent line and a secant line. They will learn formulas and techniques to enable them to differentiate algebraic, trigonometric, inverse trigonometric, inverse, exponential and logarithmic functions. Students will apply the derivate to analyze optimization problems, related rates problems and position functions.

3) They will analyze integrals by evaluating areas under the curve. Students will use the Fundamental Theorem of calculus to evaluate Integrals using anti-derivatives. They will apply integrals to problems involving area, velocity, acceleration. Lastly, students will learn techniques to integrate using substitution.

Pacing Guide									
1st Marking Period		2nd Marking Period			3rd Marking Period		4th Marking Period		
September	October	November	December	January	February	March	April	May	June

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
Preparing for Calculus	**Limits, Graphs, and Functions**	**Differentiation**	**Applications of the Derivative**	**Exponential and Logarithmic Functions**	**Integration and its Applications**
2 weeks	6 weeks	8 weeks	8 weeks	4 weeks	5 weeks

Course Overview		
Central Understandings Insights learned from exploring generalizations through the essential questions. (Students will understand) • Formal definitions and graphical interpretations of limits and continuity • Formal definition, application and properties of a derivative. • Formal definition, application and properties of an integral. • Calculus can be used to extend our mathematical boundaries. • Calculus is the study of change.	**Essential Questions** • What is a limit and how can it be interpreted? • What is a derivative? • What is an integral? • What is Calculus?	**Assessments** • Formative Assessments • Summative Assessments

Content Outline	Standards
I. Unit 1 – Preparing for Calculus II. Unit 2 – Limits, Graphs, and Functions III. Unit 3 – Differentiation IV. Unit 4 – Applications of the Derivative V. Unit 5 – Exponential and Logarithmic Functions VI. Unit 6 – Integration and its Applications	Many standards referenced in this curriculum are from the California Calculus standards.

Calculus Standards for Mathematical Practice

The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:

Standards	***Explanations and Examples***
1. Make sense of problems and persevere in solving them.	Students solve problems involving equations and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2. Reason abstractly and quantitatively.	This practice standard refers to one of the hallmarks of algebraic reasoning, the process of de-contextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems and then transforming the models via algebraic calculations to reveal properties of the problems.
3. Construct viable arguments and critique the reasoning of others.	In Calculus, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs and tables. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Indeed, other mathematical practices in Calculus might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.
5. Use appropriate tools strategically.	Students consider available tools such as spreadsheets, a function modeling language, graphing tools and many other technologies so they can strategically gain understanding of the ideas expressed by individual content standards and to model with mathematics. For example, students can use the graphing calculator to help determine the value of a limit as you get closer and closer to a set value.
6. Attend to precision.	In Calculus, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely helps students understand the idea in new ways.
7. Look for and make use of structure.	In Calculus, students should look for various structural patterns that can help them understand a problem. For example, seeing the pattern to the power rule after using the derivative with limits (e.g., $f(x) = 3x^2$ results in $f'(x) = 6x$ after $\lim_{h\to o} \frac{3(x+h)^2-3x^2}{h}$).
8. Look for and express regularity in repeated reasoning.	Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism.

Unit 1 – Preparation for Calculus, 2 weeks top	
In this initial unit, students will study many topics that will help them prepare for the rigors of calculus. This unit will build a solid understanding of exponential and radical expressions needed for later concepts in calculus.	
Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Knowing how to operate with radicals and exponents helps manipulate expressions in Calculus.	• How do you use the exponent properties to simplify expressions?
Calculus Standards	

EXPRESSIONS, EXPONENTS, AND RADICALS

P-C.1
Use interval notation to describe sets of numbers.

P-C.2
Find rational zeros of a polynomial.

P-C.3
Add two rational expressions with literal numerators.

P-C.4
Use fractional and negative exponents to simplify rational expressions.

P-C.5
Simplify complex radical expressions.

P-C.6
Rationalize numerator and denominator of a fraction involving radicals with the use of conjugate.

Unit 2 – Functions, Graphs, and Limits, 6 weeks top	
This unit begins with the classic tangent to a curve problem by approximating secant lines that are getting closer and closer to becoming the tangent. Some practical applications of the tangent concept are explored such as the velocity problem. Students then discuss limits formally, including one sided limits and infinite limits and limits that do not exist. This is further enhanced by discussing limit laws and the precise definition of a limit. This limit concept will then lead into the topic of the derivative and rates of change.	
Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• A concept of a limit allows you to determine the value of a function by getting really close to a specified value. • The properties of limits follow many of the properties of real numbers. • The type of continuity affects the limit of a function. • Patterns can continue to infinity and yet still have a limit as to how big they can get.	• How do you determine a value of a function for a value that is restricted in the domain? • How can you determine a limit of a function from different with functions illustrating different continuities? • What are one-sided limits and infinite limits?

Calculus Standards

FUNCTIONS, GRAPHS, AND LIMITS

FGL.1
Demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity.

FGL.1a
Prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.

FGL.1a
Use graphical calculators to verify and estimate limits.

FGL.1c
Determine values of limits of a given function (e.g., a particular number, does not exist, etc.)

FGL.2
Demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.

Unit 3 – Differentiation, 8 weeks top	
This unit is introduced by calculating derivatives using formal definition of derivatives (i.e. $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$). Focus on using limit properties to simplify the limt and manipulating and simplifying complex fractions. The focus of the unit is learning and utilizing derivative formulas. These formulas include the derivatives of polynomials (simple power rule), products (product rule), quotients (quotient rule), composite functions (chain rule), and higher order derivatives.	
Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• The extension of the limit allows for the calculation of the instantaneous rate of change for any given point on a continuous function. • There exist efficient approaches to determine the derivatives of functions.	• How can calculus and the concepts of limit and continuity assist us in analyzing the rate of change for curves? • What is a derivative, how do we determine it, and how is it built from the limit? • How is it possible to find the slope of a tangent line? • How do you differentiate a polynomial function using the power rule? • How do you differentiate a product of functions using the product rule? • How do you differentiate a rational function using the quotient rule? • How do you differentiate a composition of functions using the chain rule? • How do you differentiate implicitly? • How do you differentiate a function using related rates?

Calculus Standards

THE DERIVATIVE

D.1
Demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability:

D.1a
Demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function at a particular point.

D.1b
Demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.

D.1c
Understand the relationship between differentiability and continuity.

D.2
Know the chain rule and applications to the calculation of the derivative of a variety of composite functions.

D.3
Compute derivatives of higher orders.

D.4
Find the derivatives of defined functions through implicit differentiation in a wide variety of problems.

D.5 Use differentiation to solve related rate problems in a variety of pure and applied contexts involving variables that changing in respect to time.

Unit 4 – Applications of the Derivative, 8 weeks top

Now that rules of differentiation have been developed, students can pursue the applications of differentiation in greater depth. Here students will learn how derivatives affect the shape of the graph of a function and, in particular, how to locate maximum and minimum values of a function. Many practical problems require us to minimize a cost or maximize an area or somehow find the best possible outcome of a situation. Students will be able to investigate the optimal shape of a can.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• The use of the derivative has many applications. The calculation of the derivative allows for and efficient approach to the computation of possible maximum profit, minimum amount used, etc. • The derivative can provide useful information on the graph of a function.	• What role does calculus play as a tool in science, business, and other areas of study? • How is the derivative used to solve problems involving area, velocity and acceleration? • What information can be determined from the derivative to help sketch the graph of a function?

Calculus Standards

DERIVATIVE APPLICATIONS

D-A.1
Use differentiation to sketch, by hand, graphs of functions. Identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.

D-A.2
Use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.

D-A.3
Use calculus to solve business and economics problems (e.g., price elasticity of demand, maximum revenue and profit, minimum average cost).

D-A.4
Know how asymptotes are related to an infinite limit.

Unit 5 – Derivatives of Exponential and Logarithmic Functions, 4 weeks top	
In this unit, students will investigate how to determine the derivative of an exponential and a logarithmic function. Initially, the students graph exponential and natural exponential functions, thus allowing to define *e* from the limit. The students then take the derivative of an exponential function through the limit definition. Exponential functions lead students to graph logarithmic functions. Lastly, students take the derivatives of logarithmic functions by using implicit differentiation. These differentiation techniques are then used to solve growth and decay problems.	
Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• The use of the derivative has many applications. The calculation of the derivative allows for and efficient approach to the computation of possible maximum profit, minimum amount used, etc.	• What role does calculus play as a tool in science, business, and other areas of study? • How is the derivative used to solve problems involving growth and decay?
Calculus Standards	
DERIVATIVES OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS **D-ELF.1** Graph exponential function $f(x) = a^x$ and $f(x) = e^x$. **D-ELF.2** Calculate derivatives of exponential functions. **D-ELF.3** Graph logarithmic function $f(x) = \ln x$ and use it to solve exponential and logarithmic equations. **D-ELF.4** Calculate the derivatives of logarithmic functions.	

Unit 6 – Integration and its Applications, 5 weeks top	
This unit begins by calculating the area under a curve using geometry formulas (areas of a circle, semicircle, rectangle, square, triangle, trapezoid, etc.) and an understanding that the area under the *x*-axis is negative and the area above the *x*-axis is positive. This process is then applied to approximate an integral along with defining the limits of the integral (i.e., the lower bound b and the upper bound a of $\int_a^b f(x)\,dx$) and their properties. Finally, using the Fundamental Theorem of Calculus, integrals are derived using anti-derivatives and the formula for the anti-derivative of a polynomial.	
Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• The integral is the area under the curve. • The Fundamental Theorem of Calculus is a theorem that links the concept of the derivative of a function with the concept of the integral.	• What is an integral (definite and indefinite), how can it be determined and/or evaluated? • How is it possible to find the area under a curve? • What is the notation for the integral? • How do you find the integral? • What does the integral represent? • How is integration related to differentiation through the Fundamental Theorem of Calculus?
Calculus Standards	
INTEGRALS **I.1** Find the anti-derivative of a function. **I.2** Demonstrate knowledge of the Fundamental Theorem Of Calculus and use it to interpret integrals as anti-derivatives. **I.3** Use the general power, the exponential rule, and log rule to calculate anti-derivatives (indefinite integrals). **I.4** Know the definition of the definite integral. **I.5** Evaluate definite integrals and apply the Fundamental Theorem of Calculus to find the area bounded by two graphs. **I.6** Apply the definition of the integral to model problems in geometry, physics, economics, and so forth.	