Mathematics

Fairfield Public Schools

Trigonometry 42

TRIGOMETRY 42
This course will focus on developing an understanding of basic trigonometry. The main topics for this course are a) triangle and angle relationships using radians and/or degrees, b) trigonometric ratios (sine, cosine, and tangent), c) the Laws of Sines and Cosines, d) solving triangles, and e) the unit circle. The application of these topics to solve real world problems will be emphasized through the course. This course will assist students in preparation for college placement tests as well as other introductory college mathematics courses.

Pacing Guide			
1st Marking Period		2nd Marking Period	
Unit 1 **Trigonometric Functions and their Inverses** 7 weeks	Unit 2 **Graphing Trigonometric Functions** 4 weeks	Unit 3 **Trigonometric Identities** 3 weeks	Unit 4 **Solving Triangles** 4 weeks

Course Overview		
Central Understandings Insights learned from exploring generalizations through the essential questions. (Students will understand that…) • Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technologies. • Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. • Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools, and technologies. • Data can be analyzed to make informed decisions using a variety of strategies, tools, and technologies.	**Essential Questions** • How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? • How are quantitative relationships represented by numbers? • How do geometric relationships and measurements help us to solve problems and make sense of our world? • How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decisions?	**Assessments** • Formative Assessments • Summative Assessments

Content Outline	**Standards**
I. Unit 1 – Trigonometric Functions and their Inverses II. Unit 2 – Graphing Trigonometric Functions III. Unit 3 – Trigonometric Identities IV. Unit 4 – Solving Triangles	Connecticut Common Core State Standards are met in the following areas: • ***FUNCTIONS*** • ***GEOMETRY***

Trigonometry 42 Standards for Mathematical Practice **The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:**	
Standards	***Explanations and Examples***
1. Make sense of problems and persevere in solving them.	In Algebra, students solve problems involving equations and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2. Reason abstractly and quantitatively.	This practice standard refers to one of the hallmarks of algebraic reasoning, the process of de-contextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems and then transforming the models via algebraic calculations (A-SSE, A-APR, F-IF) to reveal properties of the problems.
3. Construct viable arguments and critique the reasoning of others.	In Trigonometry, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Indeed, other mathematical practices in Trigonometry might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.
5. Use appropriate tools strategically.	Students consider available tools such as spreadsheets, a function modeling language, graphing tools and many other technologies so they can strategically gain understanding of the ideas expressed by individual content standards and to model with mathematics.
6. Attend to precision.	In Trigonometry, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely helps students understand the idea in new ways.
7. Look for and make use of structure.	In Trigonometry, students should look for various structural patterns that can help them understand a problem. For example, seeing how the Law of Cosines and Pythagorean Theorem have a similar structure and uses (i.e., $c^2 = a^2 + b^2 – 2abcosC$ and $c^2 = a^2 + b^2$ respectively).
8. Look for and express regularity in repeated reasoning.	Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism. For example, when comparing two similar right triangles, many students who can determine that the ratio of corresponding sides will always be equivalent. This would allow the students to build the concept of trigonometric ratios and angle measurements.

Unit 1 – Trigonometric Functions and their Inverses, 7 weeks top

The study of trigonometry is focused on the idea of similar right triangles. Trigonometric ratios can be thought of as functions of the angles. With the help of the unit circle, the angles do not need to be between 0 and 90 degrees. By extending the domains to all real numbers, these trigonometric functions are used to model circular and periodic motions.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Trigonometric functions are natural and fundamental examples of periodic functions. • For angles between 0^o and 90^o, the trigonometric functions can be defined as ratios of side lengths in right triangles. These functions are well defined because the ratios of side lengths are equivalent in similar triangles. • For general angles, the sine and cosine functions can be viewed as the *y*- and *x*-coordinates of points on circles or as the projection of circular motion onto the *y*- and *x*-axes. • Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table.	• What are the major attributes of each trigonometric function? • How do you determine the exact values of trigonometric functions? • How do you determine the approximate values of trigonometric functions? • How do you convert degree measure to radian measure? • How do you convert radian measure to degree measure? • How is right triangle trigonometry used to solve right triangles?

Common Core State Standards

FUNCTIONS

Trigonometric Functions

Define trigonometric ratios and solve problems involving right triangles.

F-TF.6
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

F-TF.7
Explain and use the relationship between the sine and cosine of complementary angles.

F-TF.8
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

F-TF.10.Fairfield
Use a calculator to approximate the values of trigonometric functions.

F-TF.11.Fairfield
Determine the approximate value of the inverse trigonometric function.

Extend the domain of trigonometric functions using the unit circle

F-TF.1.
Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.1a.Fairfield

Convert measures of angles between degrees and radians

F-TF.1b.Fairfield

Use the relationship between the radius, arc length, and measure of the central angle of a circle to solve for missing values. For example, linear speed of an object traveling along a circular path.

F-TF.2

Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.3

Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for in terms of their values for x, where x is any real number.

F-TF.4

Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Unit 2 – Graphing Trigonometric Functions, 4 weeks top

Graphs of trigonometric functions reveal the periodic nature of such functions. These graphs illustrate the various concepts of amplitude, frequency, midline, domain, range, etc. Additionally, the graphs allow the students to gain a better understanding of the inverse function and how a trigonometric ration maps to a value of an angle.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table. • For functions that map the real numbers to the real numbers, composing a function with "shifting" or "scaling" functions changes the formula and graph of the function in readily predictable ways.	• What are the basic graphical properties of sine, cosine, and tangent? • How do you determine a sinusoidal function from given data?

Common Core State Standards

FUNCTIONS

Trigonometric Functions

Model periodic phenomena with trigonometric functions

F-TF.5

Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.7

Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

F-TF.8.Fairfield

Graph all six trigonometric functions (sine, cosine, tangent, secant, cosecant, and cotangent).

Analyze functions using different representations

F-IF.1

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF1e

Graph trigonometric functions, showing period, midline, and amplitude.

F-IF1f.Fairfield

Determine the domain and range of trigonometric functions.

Unit 3 – Trigonometric Identities, 3 weeks top

This unit begins with the development of the Pythagorean identify. This equation can be proved through prior knowledge of the equation of a circle and the trigonometric definitions. From this equation, students can prove other trigonometric identities.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• A proof is a specific type of mathematical argument, which is a connected sequence of deductive, logical statements in support of or against a mathematical claim. • A proof demonstrates the truth of a statement beyond any doubt for all possible cases.	• What approaches can be used to verify an identity? • How are trigonometric identities used to solve trigonometric equations?

Common Core State Standards

FUNCTIONS

Trigonometric Functions

Prove and apply trigonometric identities

F-TF.8

Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

F-TF.9.Fairfield

Use the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

F-TF.10.Fairfield

Use the double angle formulas for sine, cosine, and tangent and use them to solve problems.

F-TF.11.Fairfield

Prove other trigonometric identities and simplify others by using the identity $\cos^2(x) + \sin^2(x) = 1$. For example, students use this identity to prove that $\sec^2(x) = \tan^2(x) + 1$.

Unit 4 – Solving Triangles, 4 weeks top

In this last unit, students will focus on using the laws of Cosine and Sine to solve problems. Using these laws allow the student to solve problems that involve non-right triangle situations. This allows the student to then solve area problems using Heron's formula and A = 1/2absin(θ).

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• The Laws of Cosine and Sine provide a tool to solve problems involving non-right triangles.	• How are Law of Sines and Law of Cosines used to solve triangles? • How are Sines and Heron's formulas used to determine area of triangle?

Common Core State Standards

GEOMETRY

Similarity, Right Triangles, and Trigonometry

Apply trigonometry to general triangles

G-SRT.9

Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.11

Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

G-SRT.12.Fairfield

Determine the area of a sector of a circle in with an angle in radian measure.