

Name: _____

Algebra 32 Final Exam Review A

The exam is cumulative.

There are multiple choice, short answer, and open-ended problems.

Topics:

Functions and Polynomials

Exponents, Radicals & Exponential Equations

Trigonometry

Probability

Statistics

Sequences & Series

Formulas:

$$Z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$

$$p = \frac{2\pi}{\omega} \quad \omega = \frac{2\pi}{p}$$

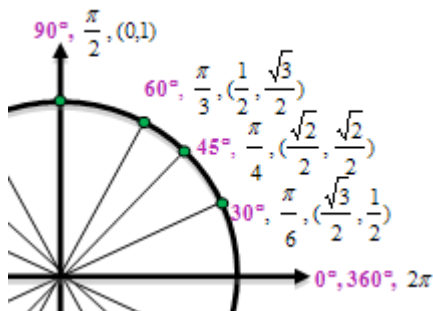
$$a_n = a_1 + (n-1)d$$

$$a_n = a_1(r)^{n-1}$$

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{a_1}{(1-r)}$$



Also: normal probability tables will be provided for the final exam

Functions and Polynomials:

- Vocabulary:
 - Leading Coefficient of a Polynomial
 - Degree
 - Zeros = x-intercepts = roots = solutions
 - Imaginary and complex numbers
- Skills:
 - Find values of polynomials by substitution or with graphs
 - Add/Subtract/Multiply/Divide polynomials
 - Long or Synthetic Division
 - Factor polynomials:
 - Always start by looking for a GCF
 - Next, look for perfect squares
 - Lastly, use the box method or grouping where appropriate
 - Always re-check your factors to see if they can be factored further
 - Transformations on Functions
 - Translations (left, right, up, down)
 - Reflections (over x-axis or y-axis)
 - Dilations (vertical stretch or shrink)
 - Inverses of Functions
 - switch the x and y and solve for y

1) Without a calculator, what is the value of the function when $x = -2$? $f(x) = x^3 + 5x^2 - 3x - 15$

2) Without a calculator, what is the value of the function when $x = -1$? $f(x) = 2x^4 - x^3 + 4$

3) Factor completely: $x^3 + 5x^2 - 3x - 15$

4) Factor completely: $x^3 - 3x^2 + 4x - 12$

5) Factor completely: $x^2 + x - 30$

6) Factor completely: $2x^2 - 13x + 15$

7) Factor completely: $2x^4 + 16x^2 + 24$

8) Factor completely: $x^4 - 7x^2 + 10$

9) Factor completely: $x^4 - 7x^2 - 18$

10) Add the following: $(2x^4 + 2x^3 - 2x^2) + (x^4 - 3x^3 + 4)$

11) Subtract the following: $(x^3 - 5x^2 + 10) - (2x^3 - 3x^2 + 1)$

12) Subtract the following: $(5x^2 + 6x - 8) - (x^3 + 2x^2 + 4x - 6)$

13) Multiply the following:

a. $(x^2 + 1) \cdot (x^2 - 2x - 7)$

b. $(x^2 - 4) \cdot (x^2 + 4)$

c. $(2x - 3) \cdot (x^2 - x + 5)$

d. $(x + 5) \cdot (x^2 + 2x - 1)$

14) Divide using long or synthetic division:

a. $(x^2 + 5x - 14) \div (x - 2)$

b. $(x^2 - 2x - 48) \div (x + 5)$

c. $(x^3 + x + 30) \div (x + 3)$

15) Perform the operations on complex numbers. Make sure to simplify your answer completely.

a. $i =$

b. $i^2 =$

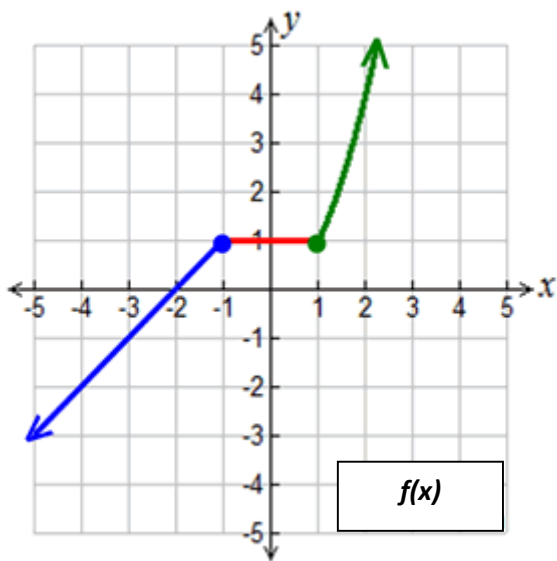
c. $(7 - 5i) + (3 - 2i)$

d. $(4 + i) - (6 - 5i)$

e. $(3 + 2i)(5 - 3i)$

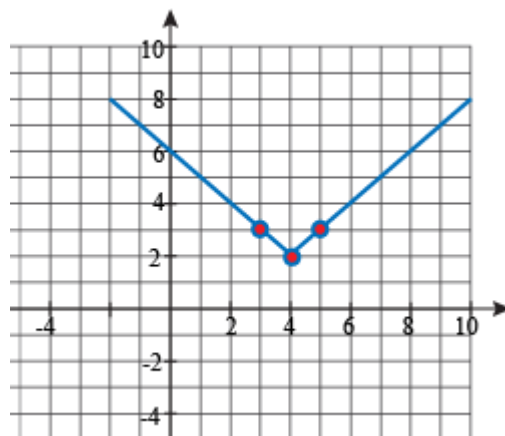
f. $(4 - i)(2 + 7i)$

16) Use the function below to answer the following questions:



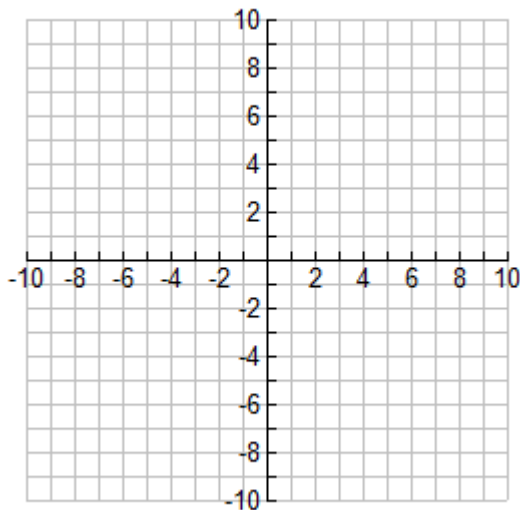
- a) What is the y-intercept? _____
- b) What is the x-intercept? _____
- c) Estimate the following using the graph:
 - i. $f(1)$
 - ii. $f(-3)$
 - iii. $f(x) = -2$
 - iv. $f(x) = 4$
 - v. $f(x) = 1$

17) Transform the function at right using the rule: $f(x + 2) - 5$



18) A polynomial function has x-intercepts at -5, 0 and 3.

- a. Write an equation that has the correct x-intercepts: $f(x) =$ _____
- b. Sketch a function that has the x-intercepts listed above:



19) Determine the inverses of the following functions:

a) $f(x) = \sqrt{x + 3}$

b) $y = x + 8$

c) $f(x) = \sqrt[3]{x - 1}$

d) $y = 3x - 12$

Exponents, Radicals & Exponential Equations

- Skills:
 - Switch between radical and rational exponent form
 - Use properties of exponents and radicals to simplify expressions
 - Identify and graph exponential growth/decay functions
 - General Exponential Form For Graphing: $y = a \cdot b^x$
 - Exponential Growth: $A = P(1 + r)^t$
 - Exponential Decay: $A = P(1 - r)^t$
- Vocabulary:
 - Initial Amount
 - Growth Rate

1) Simplify: $x^{\frac{2}{7}} \cdot x^{\frac{1}{2}} =$

2) Simplify: $\left(x^{\frac{2}{7}}\right)^7 =$

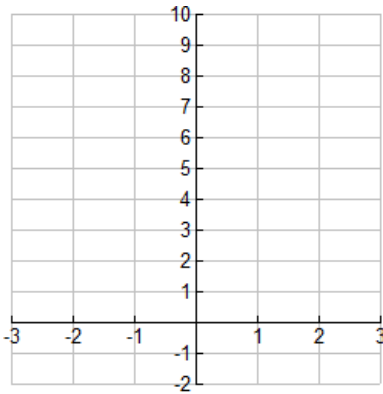
3) Re-write in radical form: $x^{\frac{5}{6}}$

4) Re-write in exponent form: $\sqrt[3]{x^2}$

5) Simplify, then re-write in radical form: $x^{\frac{1}{4}} \cdot x^{\frac{3}{8}} =$

- 6) The average weight W (in kilograms) of Pacific Salmon can be modeled by $W = 0.48(1.52)^x$ where x is the age in years.
- What is the initial weight of the cod?
 - What is the growth rate of the salmon per year?
 - What would the estimated weight of the cod be at age 5 years? Round to 2 decimal places.
 - When will the cod reach 12 kilograms? Round to 2 decimal places.

7) Graph the function $f(x) = 2^x$ using the table.



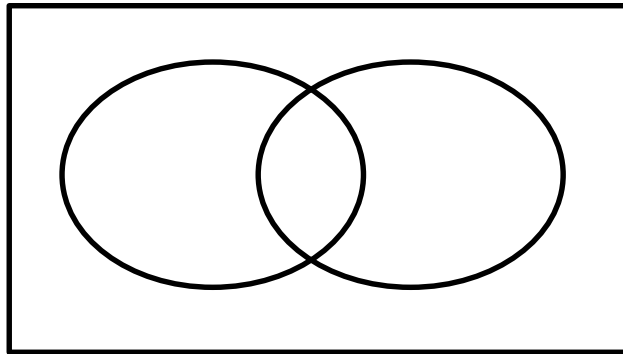
x	y
-2	
-1	
0	
1	
2	
3	

Probability:

- Basic probability: with and without replacement
- Symbols/Notation:
 - U = union
 - \cap = intersection
 - A^C = complement of A = not A
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Conditional Probability = $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- 2 main ways to show Independence:
 - $P(A) \cdot P(B) = P(A \cap B)$
 - $P(A|B) = P(A)$
- Venn Diagrams
- 2-Way Tables

1) Draw and label a Venn Diagram for the following information, then use it to answer the questions.

- There are a total of 150 children at a summer camp
- A = likes chocolate = 80 children
- B = likes ice cream = 120 children
- $A \cap B$ = likes both = 70 children

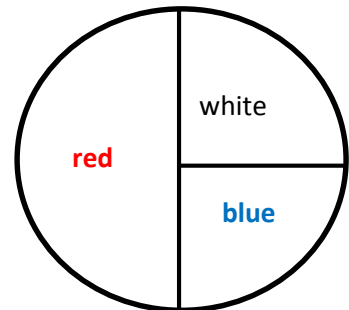


- a) What is the value of $A \cup B$?
- b) What does $A \cup B$ mean in context of the problem?
- c) What is A^C ? (give the value and context)
- d) What is B^C ? (give the value and context)

- 2) A laundry basket contains 13 socks: 7 black, 4 white and 2 pink.
- You take 1 sock from the basket, replace it because you can't find the match, and then take another. Find the probability the 1st sock was black, and the second was pink.
 - You take 1 sock from the basket, and then take another without replacing the 1st. Find the probability the 2nd sock is black given that the 1st is white.
- 3) In one group of 100 students, 50 are taking AP classes, 60 are taking music classes, and 35 are taking both music and AP classes. What is the probability that a student from this group is taking music or AP classes? (*Hint: use formulas from box at the beginning of this section*)

4) You roll a die (with 6 sides) **and** spin the spinner shown below.

- What is the probability the die lands on 2 and the spinner lands on red?
- What is the probability the die lands on 6 and the spinner lands on white?



- You only spin the spinner: what is the probability it does not land on not red?
- You only roll the die: what is the probability it is a number less than 4?

5) A survey of high school students asked: What is your favorite pet? The results are summarized below:

	Dogs	Cats	Other	TOTAL
9 th	130	60	10	200
10 th	195	90	15	300
11 th	228	105	17	350
12 th	98	45	7	150
TOTAL	651	300	49	1000

Using these 1000 students as the sample space, a student from this study is randomly selected.

- What is the probability of selecting a student whose favorite animal is a cat?
- What is the probability of selecting a 9th grade student?
- What is the probability that a student is a 10th grader and likes cats?
- What is the probability that the student prefers dogs, given that the student is a 9th grader?
- What is the probability that the student is a 12th grade student, given that the student prefers dogs?
- Are liking dogs and being a 9th grader independent? Use formulas and show work to explain your answer.

Statistics:

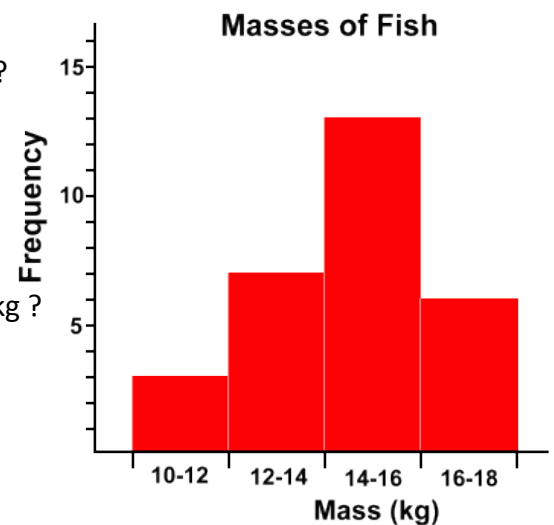
- Vocabulary: population, sample, census, observational study, experiment, survey, histogram, frequency, relative frequency, mean, standard deviation
- Parameters v. Statistics
 - Parameter is from the population
 - Statistic is from the sample
- 68-95-99.7% Rule
- $z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}$
- Find probabilities for z's using the table and calculator.

- 1) Briefly explain the difference between a sample and a census.

- 2) Briefly explain the difference between an experiment and an observational study.
- 3) 500 Fairfield residents were surveyed and asked if they exercise at least once a week. 80% of the people surveyed said they exercise every day. Nationally, the percentage of people who say they exercise at least once a week is 73%.
- Describe the population:
 - Identify the value of the parameter:
 - Describe the sample:
 - Identify the value of the statistic:
- 4) A census was taken on a college campus and found that 79% of the students have a smartphone. A sample of 20 students was taken in the cafeteria and only 60% of those students had a smartphone. Identify the parameter and statistic.
- Describe the population:
 - Identify the value of the parameter:
 - Describe the sample:
 - Identify the value of the statistic:

5) Use the histogram at right to answer the following questions:

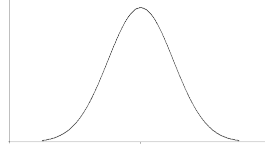
- How many fish were weighted to make this histogram?
- What is the frequency for fish weighing 10-12kg ?
- What is the relative frequency for fish weighing 10-12kg ?
- What does the height of each bar represent?
- What is the area of the bar representing fish that weigh 16-18kg?



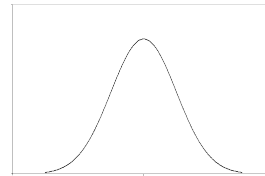
- According to the graph of 'Masses of Fish', approximately what percent of fish weight between 12 and 16 kg?

6) Shade the area you are being asked to find, then find it using your z-tables.

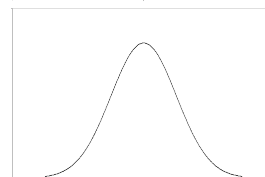
a. What is the area to the left of $z = 1.32$?



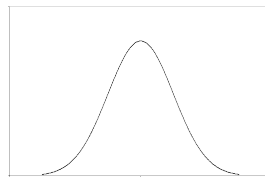
b. What is the area to the right of $z = 1.32$?



c. What is the area to the left of $z = 2$?



d. What is the area between $z = -1.01$ and $z = 0.52$?



7) The mean of a normally distributed data set is 25, with a standard deviation of 4.

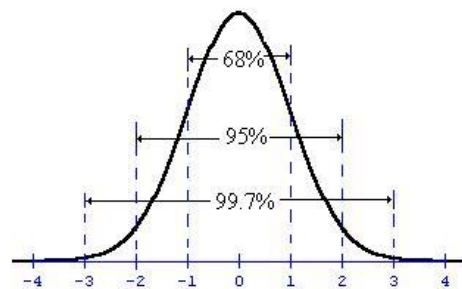
a. Approximately 50% of the data falls below what number?

b. Approximately 68% of the data fall between what two numbers?

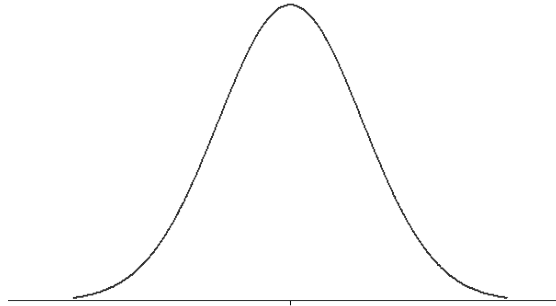
c. Approximately 95% of the data fall between what two numbers?

d. Approximately 99.7% of the data fall between what two numbers?

e. Approximately 16% of the data fall above what number?

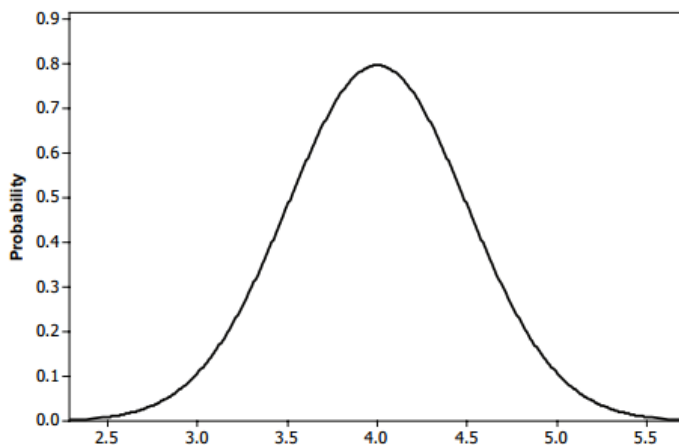


- 8) The mean height of kindergarteners is 40 inches with a standard deviation of 2 inches.
- Label the normal curve for 1, 2, and 3 standard deviations above and below the mean.



- What percentage of kindergarteners are between 36 and 40 inches tall?
- What percentage of kindergarteners are above 42 inches tall?
- What percentage of kindergarteners are below 44 inches tall?
- 69% of kindergarteners are between what heights?
- A kindergartener is 41 inches tall:
 - What is the z-score for this child?
 - What is the probability kindergartener is 41 inches or less in height?

- 9) Use the graph of the normal curve below to determine the mean and the standard deviation.



Mean = _____

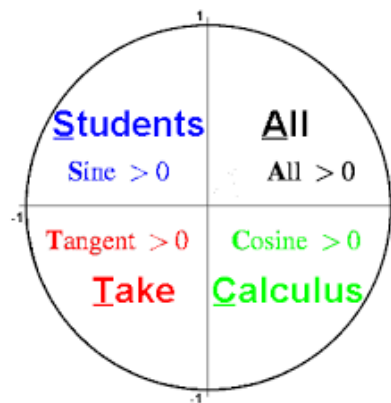
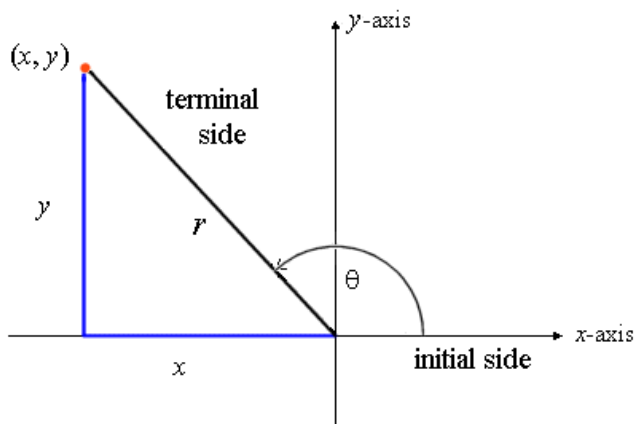
Standard Deviation = _____

Trigonometry:

- $\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$
- Sinusoidal function: $y = A\sin(\omega x) + k$
 - Amplitude
 - Period
 - Vertical shift up/down
- Radians to degrees: multiply an angle in radians by $\frac{180}{\pi}$ to convert to degrees.
- Vocab: Reference angle, co-terminal angles
- Signs of Trig Functions in different Quadrants

	quadrant			
	1	2	3	4
sin	+	+	-	-
cos	+	-	-	+
tan	+	-	+	-

II Sin positive	I All positive
III Tan positive	IV Cos positive



- 1) Find the exact value of $\sin 150^\circ$
- 2) Find the exact value of $\cos 240^\circ$
- 3) Find the exact value of $\tan 135^\circ$

- 4) Determine the exact value of $\sin \frac{\pi}{3}$
- 5) Determine the exact value of $\cos \frac{\pi}{4}$
- 6) Find the exact value of $\sin 240^\circ$
- 7) Find the exact value of $\cos 225^\circ$
- 8) Find the exact value of $\tan 120^\circ$

- 9) Determine the exact value of $\sin \frac{3\pi}{4}$
- 10) Determine the exact value of $\tan \frac{\pi}{6}$

Hint: Draw a diagram to help you for #11-13.

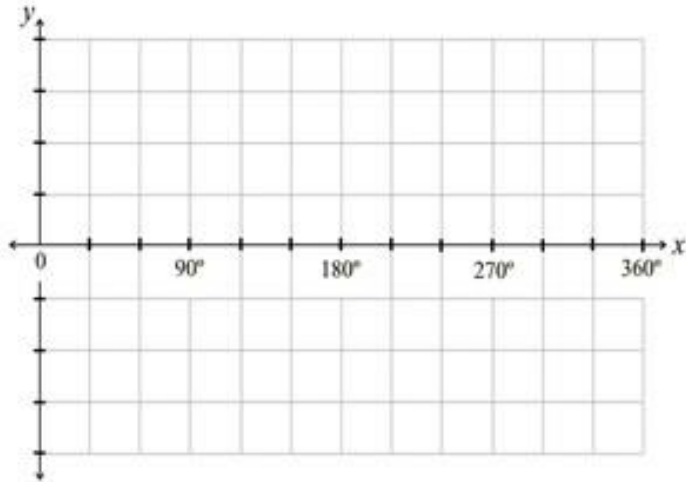
- 11) Given $\tan \theta = \frac{3}{4}$ and $180 < \theta < 270$, find $\sin \theta$.

- 12) Given $\cos \theta = \frac{1}{2}$ and $0 < \theta < 90$, find $\sin \theta$.

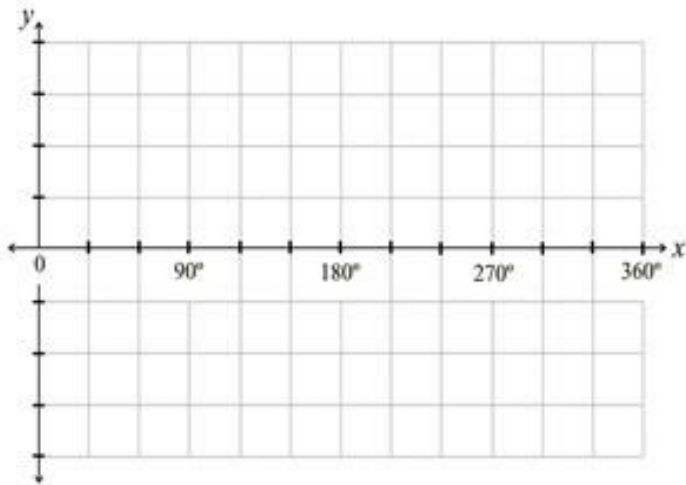
- 13) Given $\sin \theta = \frac{1}{2}$ and $90 < \theta < 180$, find $\cos \theta$.

14) Graph 1 period of the functions below. Remember to label your scale and axes.

a. $f(x) = \sin\theta$

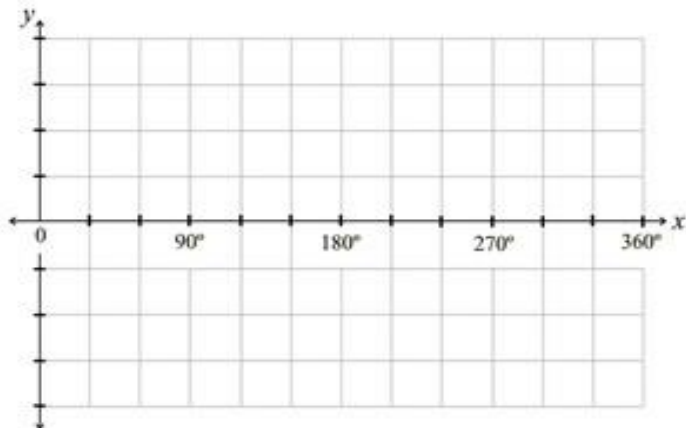


b. $f(x) = \cos\theta$



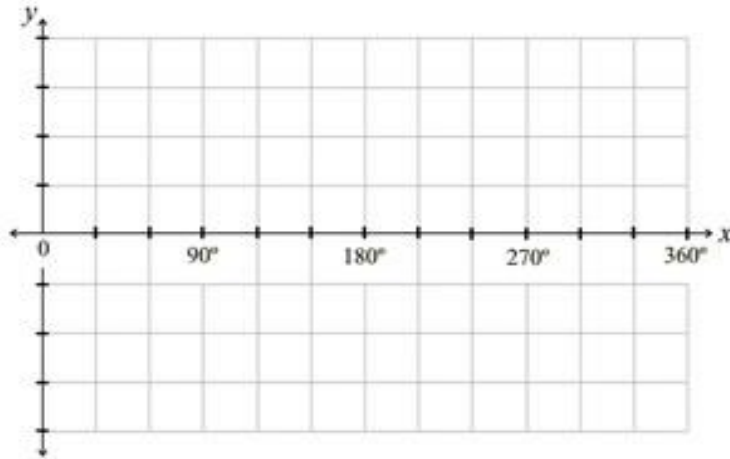
15) $f(x) = 3\cos(2x)$

- a. What is the amplitude?
- b. What is the period?
- c. What is the horizontal shift?
- d. What is the vertical shift?
- e. Identify any other transformations on the curve.
- f. Graph one period of the function.



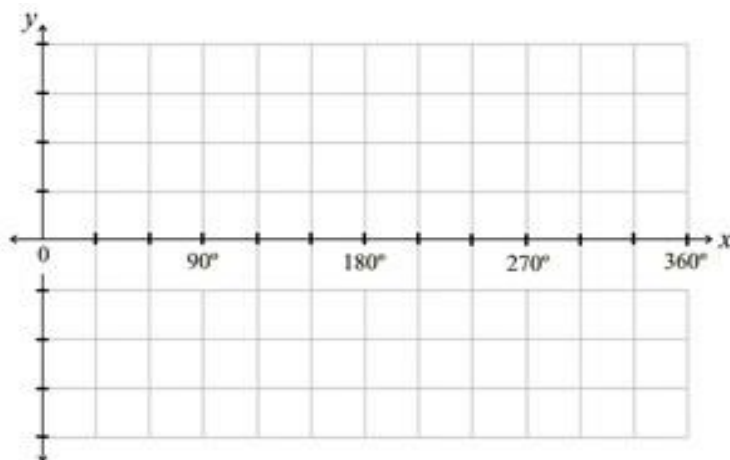
16) $f(x) = -4\sin(x) + 2$

- What is the amplitude?
- What is the period?
- What is the horizontal shift?
- What is the vertical shift?
- Identify any other transformations on the curve.
- Graph one period of the function.



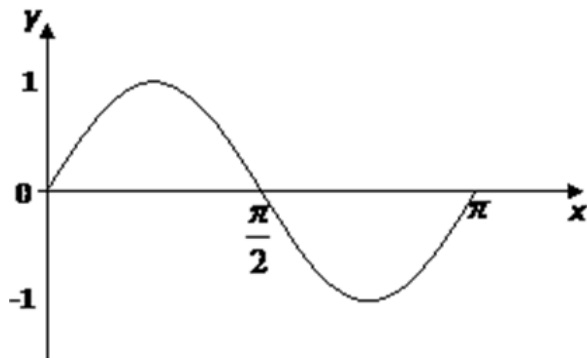
17) $f(x) = 2\cos(3x) - 1$

- What is the amplitude?
- What is the period?
- What is the horizontal shift?
- What is the vertical shift?
- Identify any other transformations on the curve.
- Graph the sinusoidal function.

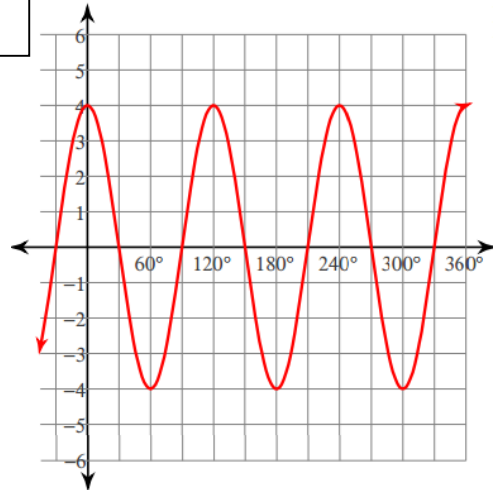


18) Write the equation of the sinusoidal functions below:

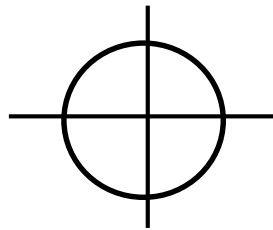
a.



b.



19) Convert 330° to radians and draw it in the correct quadrant. What is the reference angle?



$$\frac{\pi}{180^\circ}$$

Radian measure: _____

Reference angle: _____

What quadrant is it in? _____

20) Find a positive and a negative co-terminal angle to 150°

Sequences & Series: Sections 12.1-12.4

- An **Arithmetic Sequence** has a common difference between terms, while a **Geometric Sequence** has a common ratio between terms.
- a_1 is the 1st term in a sequence and a_n is the general notation for the last term.
- When finding the sum of the series below, the number 4 tells you how many terms to add up.

$$\sum_{n=1}^4 (n + 8)^2$$

1) Determine if each sequence is Arithmetic, Geometric, or neither. Find the 8th term in the sequence. Write the rule for the nth term in simplest form.

a. 3, 5, 7, 9, ...

a) Type of sequence: _____

b) 8th term: _____

c) Rule: _____

d) 40th term: _____

b. -2, 1, 4, 7, ...

a) Type of sequence: _____

b) 8th term: _____

c) Rule: _____

d) 60th term: _____

2) Compute the sum of each of the series below.

a) $\sum_{i=1}^3 2i - 1$

b) $\sum_{i=1}^4 \frac{1}{i - 4}$

c) $\sum_{n=1}^{\infty} 3 \left(\frac{1}{2}\right)^{n-1}$

3) Find the 1st 5 terms of the sequence $a_n = -1(n + 5) - 1$