Precal 41 Name: ____________________

Final Review A Date: ____________

Rationals

1. Solve the following radical equations. Don't forget to check for extraneous solutions.

   $2x = \sqrt{100-12x} - 2$

2. Solve: $\sqrt[3]{(x-5)^2} + 14 = 50$

3. Find ALL zeros. (real and/or complex) $f(x) = x^5 - 18x^3 + 30x^2 - 19x + 30$

4. Write a polynomial with least degree that has the following zeros. -3, 1 (multiplicity 2), 4i

5. Is (x-2) a factor of $x^3 + 3x^2 + 5x - 30$?

6. Find the x-asymptote(s), y-asymptote, oblique asymptote, x-intercept(s), y-intercept, and/or hole(s) for the following functions.

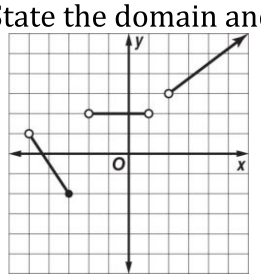
   a. $f(x) = \dfrac{x^2+2x-3}{x+2}$ b. $f(x) = \dfrac{x-2}{x^2-6x+8}$

7. Find the end behavior of $f(x) = 4x^3 - 5x^2 + 2x + 3$.

   $\lim_{x\to\infty} g(x) =$

   $\lim_{x\to-\infty} g(x) =$

8. State the domain and range of the function shown



9. Find the inverse of $f(x) = \frac{3x}{x-2}$.

10. Find the domain of the following functions.

   a. $y = \sqrt{x+3}$ b. $f(x) = \dfrac{2}{x^3 - 3x^2 - 10x}$

Exponential/Logarithmic

11. Condense: $2 \log x - \log 3$

12. Expand: $\log_9 \frac{x^2}{13y^5}$

13. Solve:

a. $8^{2x+3} = \left(\frac{1}{4}\right)^{x+1}$

b. $\log_2 x^3 = 6$

c. $\log_4(2x) + \log_4(x - 2) = 2$

14. Evaluate: $\log_{16} \frac{1}{4}$

15. Suppose $1750 is put into an account that pays an annual rate of 4.25% compounded weekly. How much will be in the account after 36 months?

16. A scientist has 37 grams of a radioactive substance that decays 30% continuously. How many grams of radioactive substance remain after 9 years?

Polar

17. Find the rectangular coordinates of:

a. $(4, 120°)$

b. $(-2, 3\pi/4)$

c. $(3, -\pi/3)$

18. Find one set of polar coordinates for the following rectangular coordinates if $r > 0$:

a. (3, 6)

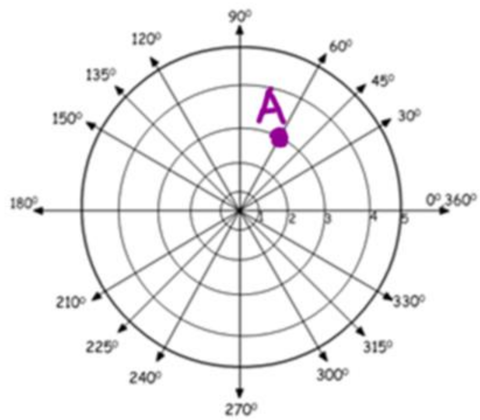
b. (-2, 7)

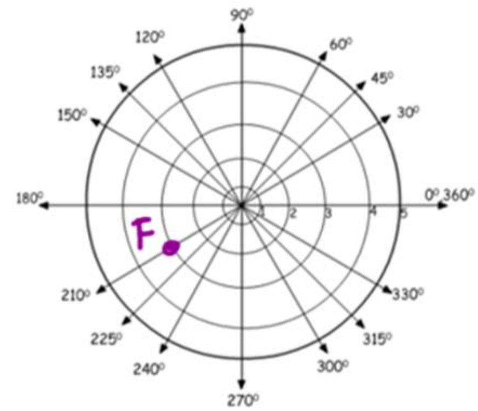
c. (-1, -7)

19. Name the polar coordinates of points A and F graphed below if:

a. $r>0$ and $0 \leq \theta \leq 360°$

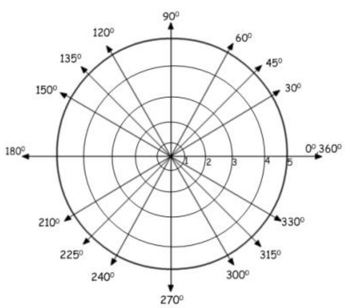
b. r<0 and $0 \leq \theta \leq 360°$

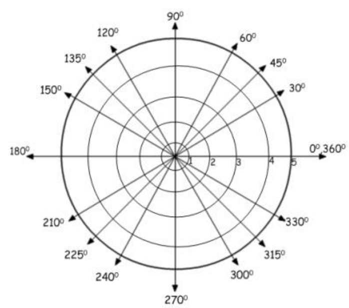


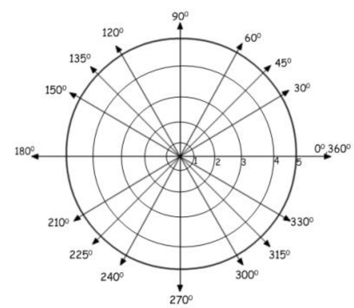


20. Graph the polar equations:

r = 3 + 3sinθ b. $\theta = -\pi/6$ c. r = 5cos θ





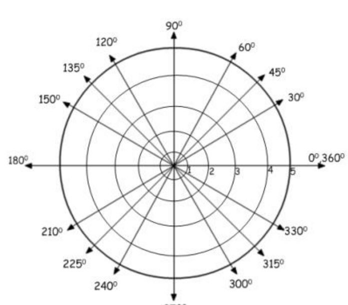


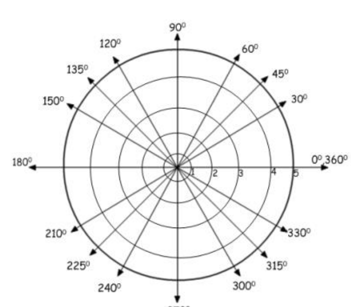
21) Write the polar equations in rectangular form:

a. r = -6sinθ b. r = 2cosθ

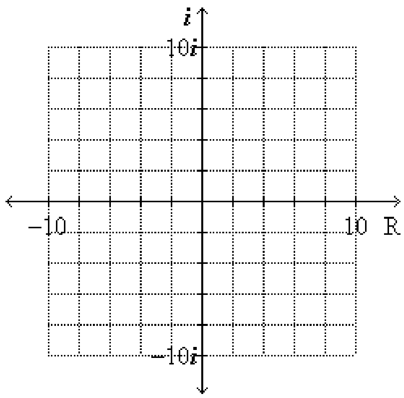
22) Write the rectangular equations in polar form, then graph:

a. $x^2 + y^2 = 16$ b. $(x-2)^2 + y^2 = 4$



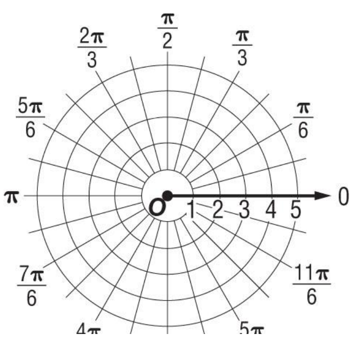


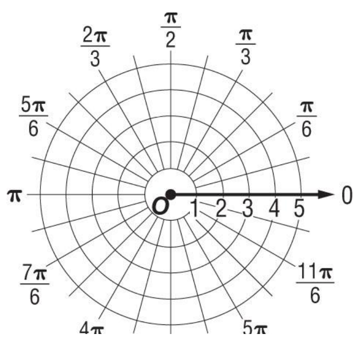
23) Graph the number $-3 + 4i$ in the complex plane and find its absolute value.

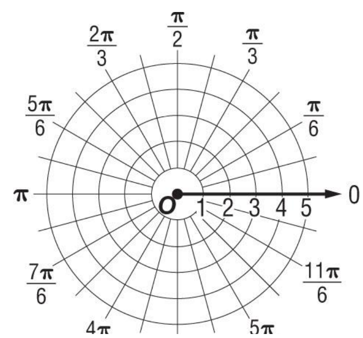


24) Graph the polar equation and state the symmetry.

a. $r = 2 - 2\sin\theta$ b. $r = 3 + 2\cos\theta$ c. $r = 3\sec\theta$







Vectors

25) Find 5**r** – 2**s** if **r** = $\langle 3, 9\rangle$ and **s** = $\langle -3, 6\rangle$

26) Find the angle between the two vectors (CALC):

a) $\langle 2, 12\rangle, \langle -3, 4\rangle$

b) $\langle -2, 5\rangle, \langle -7, -10\rangle$

27) Let $\overrightarrow{AB}$ be the vector with initial point $A(10, -4)$ and terminal point $B(-1, -3)$. Write $\overrightarrow{AB}$ as a linear combination of the vectors **i** and **j**.

28) Find the magnitude of $\overrightarrow{AB}$ with initial point $A(-3, 7)$ and terminal point $B(8, -9)$.

29) Find the component form of $\overrightarrow{AB}$ with initial point $A(-12, 7)$ and terminal point $B(8, -2)$.

30) A plane takes off at 220 miles per hour at an angle of 51° with the ground. Find the magnitude of the horizontal and vertical components of its velocity. Round to the nearest tenth.(CALC)

31) Charles leaves his apartment and walks 55° north of west for 1000 feet and then walks 300 feet due north to go bowling. How far and at what quadrant bearing is Charles from his apartment? (CALC)

Parametric

32) Write the following parametric equations in rectangular form:

a) $x = 3t - 1, y = 2t^2 + 6$

b) $x = 4\cos\theta \qquad y = 2\sin\theta$

33) Suppose Mr. Shanazu hit a golf ball with an initial velocity of 150 feet per second at an angle of 30° to the horizontal. Round all answers to the *nearest hundredth.* (CALC)

a) Write a set of parametric equations that describe the position of the ball as a function of time.

b) How long is the golf ball in the air?

c) When is the ball at its maximum height?

d) What is the maximum height of the golf ball?

e) His goal was to hit the golf ball at least 600 feet. Did he reach his goal? How far away did the golf ball land?

Trigonometry

34. A point (6, 8) is on the terminal side of angle θ. Find the exact value of the $\cos\theta$.

35. A point (21, 28) is on the terminal side of angle θ. Find the exact value of the $\csc\theta$.

36. A point (2, -3) is on the terminal side of angle θ. Find the exact value of the $\sin\theta$.

37. Find the exact value of $\cos 75^{o}$.

38. Find the exact value of $\tan 15^{o}$.

39. Name the quadrant in which the angle θ lies if:

a. $\cos\theta < 0$, $\csc\theta < 0$ _______________________

b. $\cot\theta < 0$, $\cos\theta > 0$ _______________________

c. $\sec\theta < 0$, $\tan\theta < 0$ _______________________

d. $\sin\theta > 0$, $\cos\theta > 0$ _______________________

40. Find the exact value of the 5 remaining trig functions if $\sec\theta = \frac{9}{8}$ and θ is in Quadrant 4.

$\sin\theta =$ _________ $\csc\theta =$ _________

$\cos\theta =$ _________ $\sec\theta =$ _________

$\tan\theta =$ _________ $\cot\theta =$ _________

41. Find the exact value of $\cos 2\theta$, if $\cos\theta = \frac{8}{17}$ and $\frac{3\pi}{2} < \theta < 2\pi$

42. Identify the amplitude and period given the equation: $y = -3 \sin 5x$

Amplitude = ____________ Period = ______________ Phase Shift = ______________

43. Identify the amplitude and period given the equation: $y = -5 \cos(4x + \pi)$

Amplitude = _____________ Period = ______________ Phase Shift = ______________

44. Identify the amplitude and period given the equation: $y = 4\cos(x - \frac{\pi}{2})$

Amplitude = _____________ Period = ________________ Phase Shift = ______________

45. Write the equation of a sine function with the given characteristics:

Amplitude = 4 Period = 3 Equation: ______________________________________

46. Write the equation of a sine function with the given characteristics:

Amplitude = 3 Period = 4π Phase Shift = $\frac{-\pi}{4}$

Equation: ______________________________________

47. Identify the domain & range of the inverse functions

	Domain	Range
$\sin^{-1}(x)$		
$\cos^{-1}(x)$		
$\tan^{-1}(x)$		

48. Find the exact value of the expression: $\tan^{-1}(-\sqrt{3}) =$ _______________

49. Find the exact value of the expression: $\sin^{-1}(\frac{\sqrt{2}}{2}) =$ ______________

50. Find the exact value of the expression: $\cos^{-1}(\frac{-\sqrt{2}}{2}) =$ ______________

51. Find the exact value of the expression: $\cos[\sin^{-1}(\frac{1}{4})] =$ ______________

52. Find the exact value of the expression: $\cos^{-1}[\cos(\frac{7\pi}{6})] =$ ______________

53. Find the exact value of the expression: $\tan[\sin^{-1}(\frac{-\sqrt{3}}{2})]$ = ________________

54. $\text{Sin}\ \theta = \frac{20}{29},\ 0 < \theta < \frac{\pi}{2},$ Find cos (2θ) ______________________________

55. Verify the identity: $\cos(\frac{\pi}{2} + \theta) = -\sin\theta$

56. Solve 6 cos x – 3 = 0 in the interval $[0, 2\pi)$

57. Solve 2sin x + 1 = 0 in the interval $[0, 2\pi)$

58. Verify the identity: $\sin^2 x \tan^2 x \csc^2 x + \cos^2 x \tan^2 x \csc^2 x = \sec^2 x$

59. Verify the identity: $\sec\theta = \sin\theta(\tan\theta + \cot\theta)$

60. Verify the identity: $\frac{\csc^2\theta - \cot^2\theta}{1 - \sin^2\theta} = \sec^2\theta$

61. Verify the identity: $1 - \frac{\sin^2\theta}{1+\cos\theta} = \cos\theta$

62. Solve in the interval $[0, 2\pi)$

a. $(\cot\theta + 1)(\csc\theta - \frac{1}{2}) = 0$

b. $\text{Cos}^2\theta - \sin^2\theta + \sin\theta = 0$

c. $2\sin^2\theta = 3(1 - \cos\theta)$

d. $\text{Cos}(2\theta) = 2 - 2\sin^2\theta$

63. What is the reference angle if $\theta = 247^0$

64. Name an angle that is coterminal with: $\frac{7\pi}{15}$

65. Two observes simultaneously measure the angle of elevation of a helicopter. One angle measured is A: 25^0 and the other is B: 40^0. If the observers are 100 feet apart and the helicopter lies over the line joining them. How far away from the helicopter are the observers A and B?

66. Solve the following triangles. Round to the nearest hundredth.

a. $a = 11cm, b = 6\ cm, A = 22^o$

b. $a = 13\ m, b = 12\ m, c = 8m$

c. $a = 9\ cm, b = 10\ cm, C = 42^o$

d. $a = 5\ cm, A = 36^o, B = 42^o$

e. $A = 63^o, a = 18in, b = 25in$

f. $A = 20^o, a = 4mm, b = 6mm$

67. Determine the area of each triangle to the nearest tenth.

a. $A = 95°,\ b = 12m, c = 18\ m$

b. $a = 44, b = 47, c = 53$

Matrices:

$A = \begin{bmatrix} -1 & 5 \\ 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 2 & -1 \\ 0 & -5 & 3 \end{bmatrix}$ $C = \begin{bmatrix} -1 & 0 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

68. Evaluate each of the following.

a. $AB + C$

b. $3AC - B$

______________________________ ______________________________

69. Solve the system of equations.

$3x - y + 2z = -3$ $\qquad$ $x =$ ______ $y =$ ______ $z =$ ______

$-x + 2y - z = 2$

$2x - 3y + z = -1$

70. Determine whether A and B are inverse matrices. Explain.

$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$

Limits and Continuity:

71. Determine whether each function is continuous at the given x-value(s). If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

a. $f(x) = \frac{x-2}{x+4}$; at $x = -4$

b. $f(x) = \frac{x+1}{x^2+3x+2}$; at $x = -1$ and $x = -2$

72. Estimate each one-sided or two-sided limit, if it exists.

a. $\lim_{x \to 0^+}(4 - \sqrt{x})$

b. $\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$

c. $\lim_{x \to -1} \frac{x+7}{x^2+8x+7}$

73. Evaluate each limit.

a. $\lim_{x \to 3}(x^2 + 3x - 8)$

b. $\lim_{x \to -6} \frac{x^2 - 36}{x + 6}$

c. $\lim_{x \to 4} \sqrt{x^2 - 2x + 1}$

Rationals:

1. x = 3
2. x = 221 , x = -211
3. x = -5, x = 2, x = 3, x = i, x = -i
4. f(x) = $x^4 + 2x^3 + 13x^2 + 32x - 48$
5. yes

6a. x-asymptote(s)_ X = -2 ____, y-asymptote_NA____, oblique asymptote_ y = x ___, x-intercept(s)_ (-3,0), (1,0)___, y-intercept _ (0, -3/2)___, hole _none____

6b. x-asymptote(s)_ X = 4 ____, y-asymptote_ y = 0 ___, oblique asymptote_ NA ___, x-intercept(s)_ none _, y-intercept _ (0, -1/4)___, hole _ (2, -1/2)___

7. $\lim_{x \to \infty} g(x) = \infty$

$\lim_{x \to -\infty} g(x) = -\infty$

8. $(-5,-3]U(-2,1)U(2,\infty)$

9. $f^{-1}(x) = \dfrac{2x}{x-3}$

10a. $(-3,\infty)$ 10b. $(-\infty,-2)U(-2,5)U(5,\infty)$

Exponential/Logarithmic:

11. $log\,\dfrac{x^2}{3}$

12. $2\log_9 x - \log_9 13 - 5\log_9 y$

13a. x = -11/8 13b. $x = \pm 4$ 13c. x = 4

14. -1/2

15. approximately $1,987.87

16. approximately 2.5 grams

Polar:

17 a. $(-2, 2\sqrt{3})$ b. $(\sqrt{2}, -\sqrt{2})$ c. $(3/2, -3\sqrt{3}/2)$

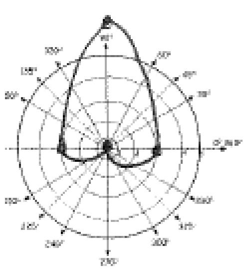
18. a. $(3\sqrt{5}, 63.4°)$ b. $(\sqrt{53}, 105.9°)$ c. $(5\sqrt{2}, 261.8°)$

19. Point A: a.(3, 60°) b. (-3, 240°) Point F: a. (3, 210°) b. (-3, 30°)

20.

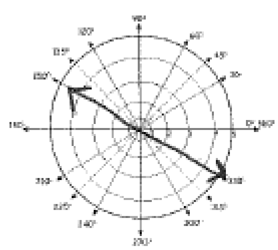
cardiod

a. $r = 3 + 3\sin\theta$



line

b. $\theta = -\pi/6$

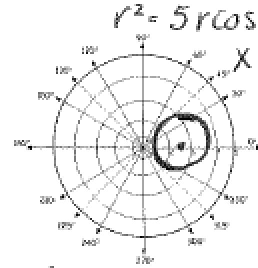


Circle

c. $r = 5\cos\theta$

$r^2 = 5r\cos\theta$

$x^2+y^2 = 5x$



$x^2 - 5x + 25/4 = 25/4$

$(x-5/2)^2 + y^2 = 25/4$

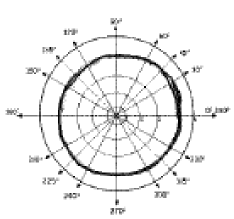
21. a. $x^2 + (y+3)^2 = 9$ b. $(x-1)^2 + y^2 = 1$

22.

a. $x^2 + y^2 = 16$

$r^2 = 16$

$r = 4$



b. $(x-2)^2 + y^2 = 4$

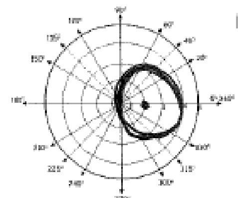
$(r\cos\theta - 2)^2 + (r\sin\theta)^2 = 4$

$r^2\cos^2\theta - 4r\cos\theta + 4 + r^2\sin^2\theta = 4$

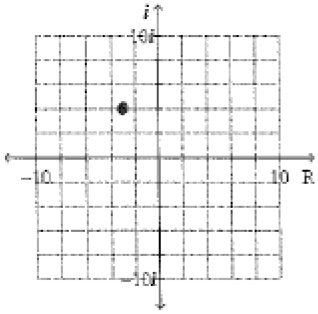
$r^2(\cos^2\theta + \sin^2\theta) = 4r\cos\theta$

$r^2 = 4r\cos\theta$

$\boxed{r = 4\cos\theta}$



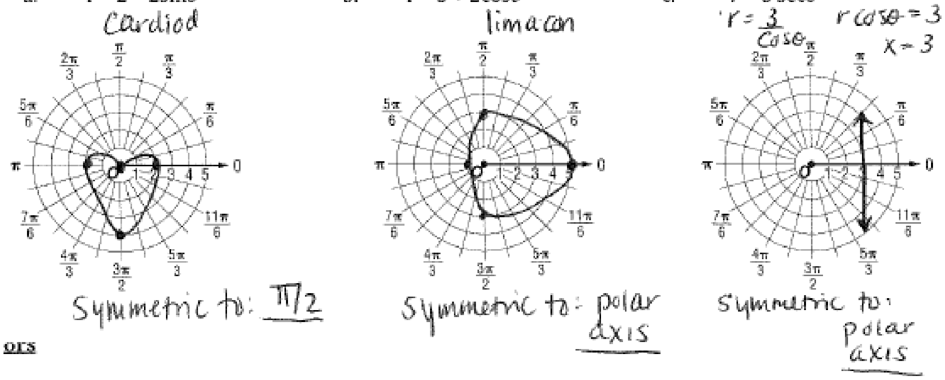
23. absolute value = 5



24.

Graph the polar equation and state the symmetry.

a. r = 2 – 2sinθ b. r = 3 + 2cosθ c. r = 3 secθ



ors

Vectors:

25. $\langle 21, 33\rangle$

26. a. 46.4° b. 123.2°

27. -11i + j

28. $\sqrt{377}$

29. $\langle 20, -9\rangle$

30. horizontal: 138.5 mph vertical: 171 mph

31. 1257.6 feet, 62.9° north of west or 27.1° west of north

Parametric:

32. a. $y = \frac{2}{9}x^2 + \frac{4}{9}x + \frac{56}{9}$ b. $\frac{x^2}{16} + \frac{y^2}{4} = 1$

33. a. $x = t150\cos 30\,, y = t150\sin 30 - \left(\frac{1}{2}\right)(32)t^2 + 0$
  b. 4.69 seconds
  c. 2.34 seconds
  d. 87.89 feet
  e. 609.2 feet, yes he reached his goal.

Trig

34. 3/5

35. $\frac{35}{28}$

36. $\frac{-3\sqrt{13}}{13}$

37. $\frac{\sqrt{6}-\sqrt{2}}{4}$

38. 2 - $\sqrt{3}$

39. a. Q3
  b. Q4
  c. Q2
  d. Q1

40. $\sin\theta = \frac{-\sqrt{17}}{9}$ $\csc\theta = \frac{-9\sqrt{17}}{17}$ $\cos\theta = \frac{8}{9}$ $\tan\theta = \frac{-\sqrt{17}}{8}$ $\cot\theta = \frac{-8\sqrt{17}}{17}$

41. $\cos 2\theta = -161/289$

42. A= 3, T = $\frac{2\pi}{5}$, PS = none

43. A = 5, T = $\frac{\pi}{2}$, PS = $\frac{\pi}{4}$ $left$

44. A = 4, T = 2π, PS = $\frac{\pi}{2}$ right

45. y = 4 sin $(\frac{2\pi}{3}x)$

46. y = 3 sin $(\frac{1}{2}x + \frac{\pi}{8})$ or y = 3 sin $(\frac{1}{2}(x + \frac{\pi}{4}))$

47.

	Domain	Range
$\sin^{-1}(x)$	[-1,1]	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1}(x)$	[-1,1]	$[0, \pi]$
$\tan^{-1}(x)$	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$

48. -60^0

49. 45^o

50. 135^o

51. $\frac{\sqrt{15}}{4}$

52. $\frac{5\pi}{6}$

53. - $\sqrt{3}$

54. $\frac{41}{841}$

55. cos $(\frac{\pi}{2} + \theta)$ = - sin θ

$\cos\frac{\pi}{2}\cos\theta$ - $\sin\frac{\pi}{2}\sin\theta$

0 – 1(sinθ)

- sin θ = - sin θ

56. $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

57. $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

58. Verify the identity: $\sin^2x \tan^2x \csc^2x + \cos^2x \tan^2x \csc^2x = \sec^2x$

$$\frac{sin^2x}{cos^2x} + 1 = sec^2x$$

$$tan^2x + 1 =$$

$$sec^2x = sec^2x$$

59. Verify the identity: sec θ = sin θ(tan θ + cot θ)

Sec θ = $\frac{sin^2\theta + cos^2\theta}{cos^2\theta}$

Sec θ = $\frac{1}{cos^2\theta}$

Sec θ = Sec θ

60. Verify the identity: $\frac{\csc^2\theta - \cot^2\theta}{1 - \sin^2\theta} = \sec^2\theta$

$$\frac{1 + cot^2\theta - \cot^2\theta}{cos^2\theta} = \sec^2\theta$$

$$\frac{1}{cos^2\theta} = \sec^2\theta$$

$$\sec^2\theta = \sec^2\theta$$

61. Verify the identity: $\frac{1+\cos - sin^2\theta}{1+\cos\theta} = \cos\theta$

$\frac{1+\cos -(1- cos^2\theta)}{1+\cos\theta} = \cos\theta$

$\frac{1+\cos -1+ cos^2\theta)}{1+\cos\theta} = \cos\theta$

$\frac{\cos\theta(1+ cos\theta)}{1+\cos\theta} = \cos\theta$

$\cos\theta = \cos\theta$

62. a. $\frac{3\pi}{4}, \frac{7\pi}{4}$
b. $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$
c. $0, \frac{\pi}{3}, \frac{5\pi}{3}$
d. No solution

63. $\theta = 67°$

64. $\theta = \frac{37\pi}{15}\ or\ \frac{-23\pi}{15}$

65. B is 46.63 feet, A is 70.92 feet

66. a) B=11.79°, C=146.21°, c=16.33cm
b) A=78.28°,B=64.67°, C=37.05°
c) c=6.87cm, A=61.19°,B=76.81°
d)C=102°,b=5.69cm, c=8.32cm
e) not a triangle
f) B=30.87°, C=129.13°,c=9.07mm OR B=149.13°, C=10.87°,c=2.20mm

67. a)107.59 m^2 b) 978.57 $unit^2$

Matrices:

68. a. $\begin{bmatrix} 3 & -27 & 12 \\ -9 & 4 & 2 \end{bmatrix}$ b. $\begin{bmatrix} 52 & -32 & 28 \\ -9 & 5 & -39 \end{bmatrix}$

69. $(1, 0, -3)$

70. Yes! $[A][B] = [B][A] = [I]$

Limits and Continuity:

71. a. No; infinite discontinuity. b. No and no; at x = -1: removable discontinuity, at x = -2: infinite disc.

72. a. 4 b. 8 c. Does not exist

73. a. 10 b. -12 c. 3