Name _____ Period _____ Date _____

MIDTERM REVIEW

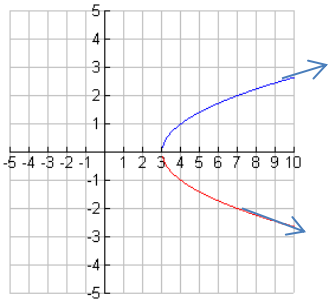
Algebra 31

Functions

1. What is the definition of a function?

2. How can you determine whether a GRAPH is a function?

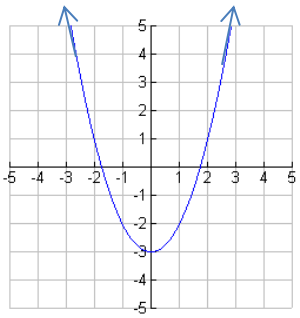
State whether the following examples are functions. Then state the domain and range. Use interval notation.

3. 

Function:

Domain:

Range:

4.) 

Function:

Domain:

Range:

5. $y = \sqrt{x - 12}$

Domain:

Range:

6. $y = \frac{1}{2x+3}$

Domain:

Range:

7. $y = -2x^2 + 6x - 25$

Domain:

Range:

For the problems below, refer to f(x), g(x) and m(x).

$$f(x) = 3x^2 - 13 \qquad g(x) = 6x + 4 \qquad m(x) = x^2 + 7x$$

8. *f(5)*

9. *m(x) = -12*

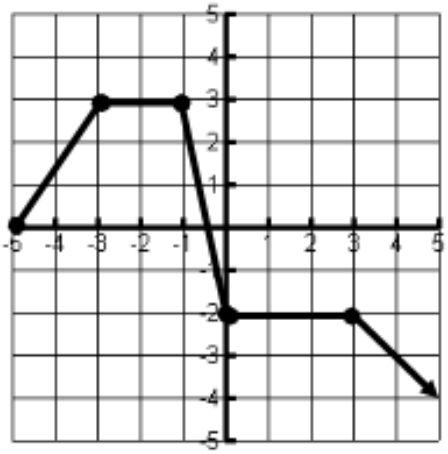
10. *g(x-7)*

11. *f(x)= 14*

12. *g[f(x)]*

13. *m[g(x)] =*

14.



a.) f(2) =

b.) f(x) = -2

c.) f(-3) =

d.) f(x) = 3

e.) f (0) =

Domain:

Range:

Increasing:

Decreasing:

Constant:

15. For *a-f*, transform each graph.

a) f(x) + 2

b) f(x – 3)

c) f(x) – 1

d) – f(x)

e) f(x + 1) – 3

f) -f(x)+2

g) State the domain:

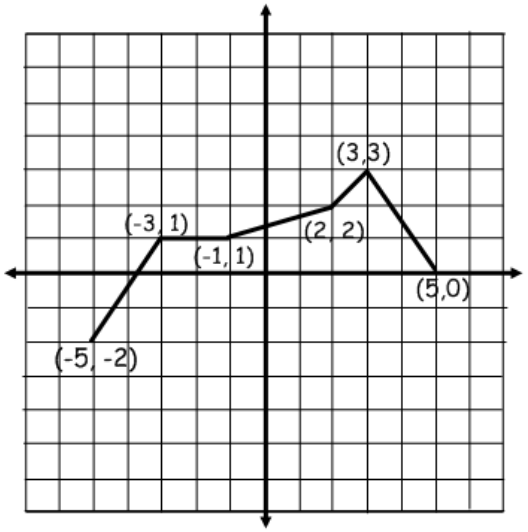
h) State the range:

i) Determine: f(3)

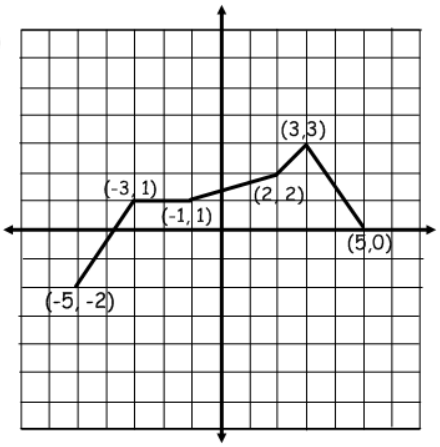
f(-5)

f(x) = 1

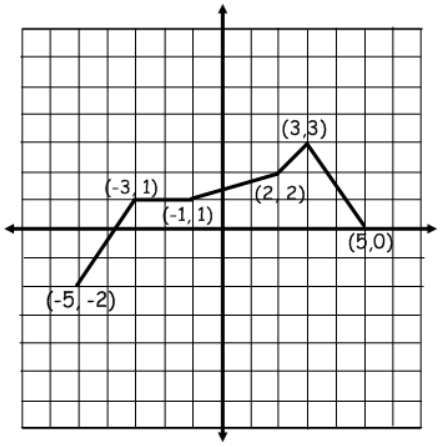
f(x)



a.)



b.)



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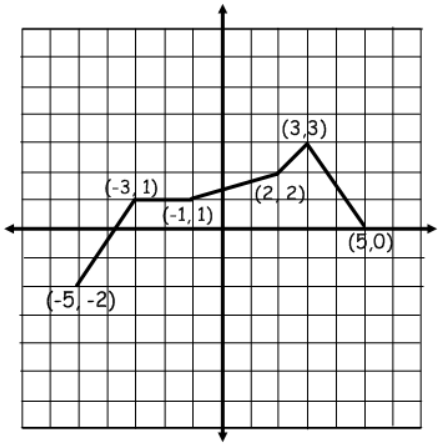
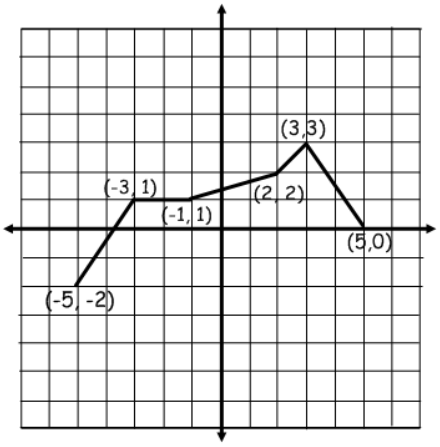
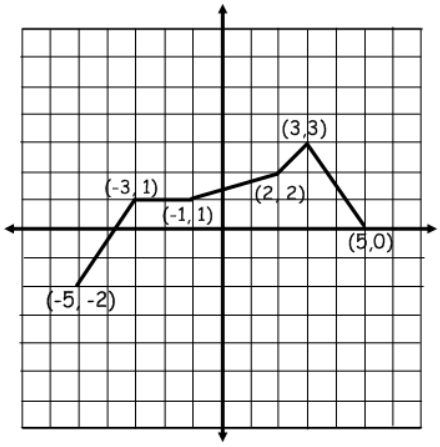


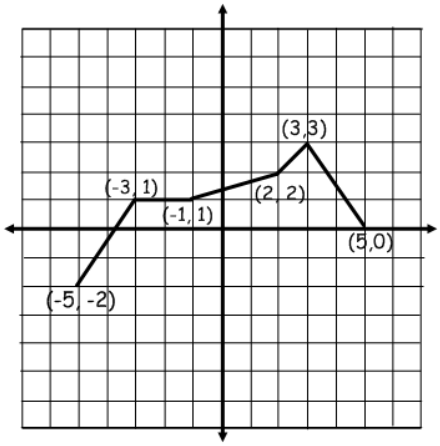
d.)



e.)



f.)



16. Give an example of a real, irrational number.

Non-Calculator

Solve:

17. Grover's Grocery Store has found that the profit, (P) from selling one type of canned soup depends on the price (x). From sales data, the model is $P(x) = -100(x-1.5)^2 + 95$. Determine the following:

a. P ($0.50) _____ b. P(x) = 70 _____

18. Given f(x) = ½ x - 4, find:

a) f(2) b) f(-4) c) f(0) d) f(6)

e) f(x) = 12 f) f(x) = - 4 g) f(x) = 3 h) f(x) = -10

19. Given $f(x) = 2x^2 + 11x - 11$, find:

a) f(- 3)

b) f(4)

c) f(0)

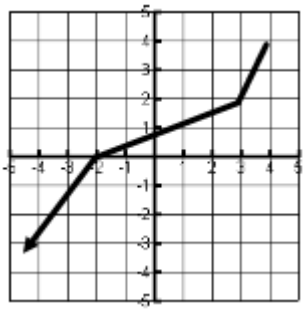
d) d) f(5)

e) f(x) = -25

f) f(x) = -23

State whether the following examples are functions. If it is, state the domain and range.

20.



Function?

Domain: _____

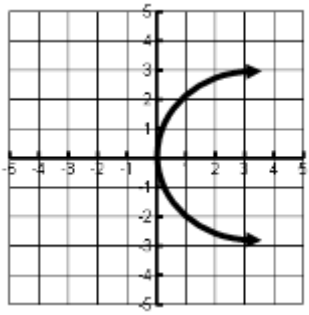
Range: _____

Increasing: _____

Decreasing: _____

Constant: _____

21.



Function?

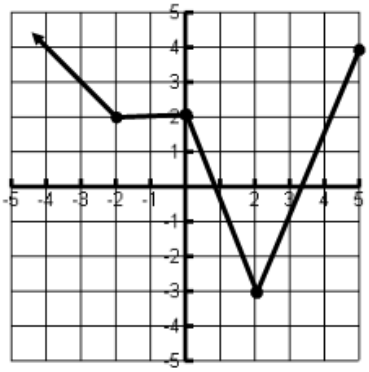
Domain: _____

Range: _____

Increasing: _____

Decreasing : _____

Constant: _____

22. 

a.) f(-2) =

b.) f (4) =

c.) f(x) = 0

d.) f(x) = 4

e.) f(x) = - 4

f.) f(2) =

Domain: _____

Range: _____

Perform the operation:

$$f(x)=\frac{3x^2+5}{2x}$$

$$g(x)=x^2+4$$

$$k(x)=2x-7$$

23. *f (5)=*

24. *g(b +2)=*

25. *k[f (-1)] =*

26. *f (0) =*

27. *g[k(x)]*

28. *k[k(10)] =*

29. Transformations. Describe in words what each transformation does.

f(x – 2) f(x + 4) f(x) – 5

_____ _____ _____

f(-x) f(x) + 1 - f(x)

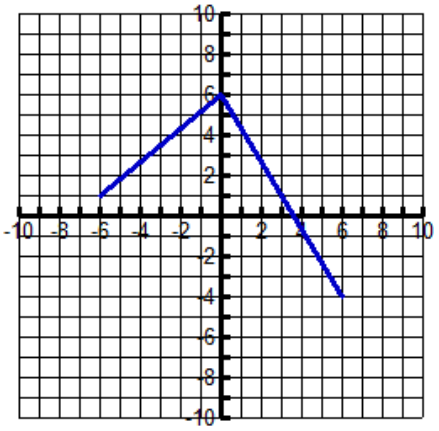
_____ _____ _____

2f(x) ½ f(x) f(x – 3)

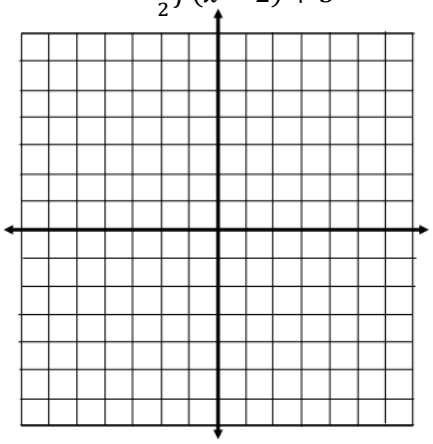
_____ _____ _____

30. Transform the graph.

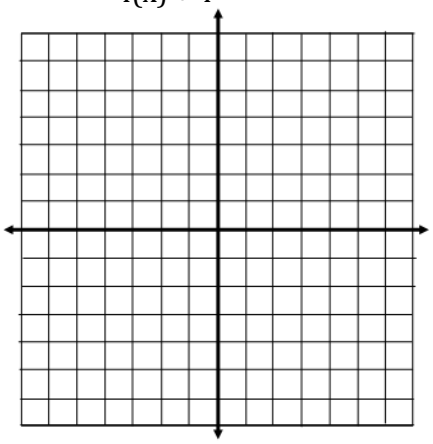
f(x)



$-\frac{1}{2}f(x-2)+3$



- f(x) + 4

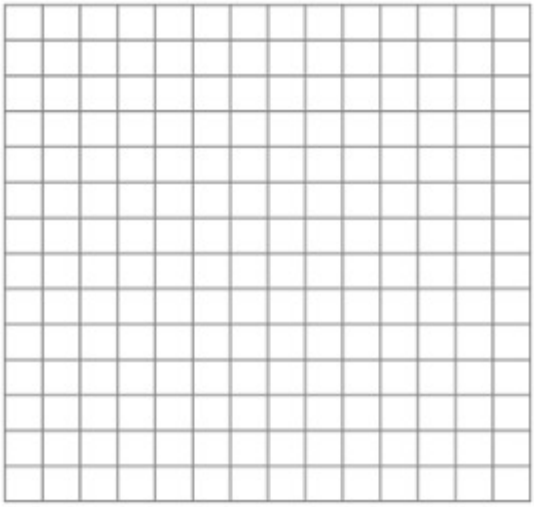


31. An American student studying abroad in Sevilla, Spain can get a cell phone through her University for $11 per month for 140 minutes or less of use. The student is charged $0.25 per minute over 140.

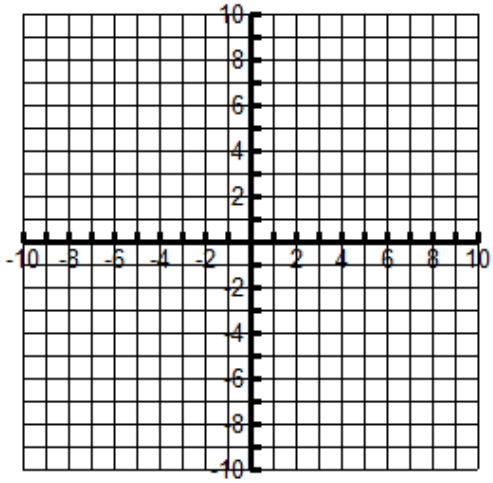
a. Determine a piece-wise function, *C(m)*, for the month cell phone bill.

b. How long did she talk on the phone if her bill was $35?

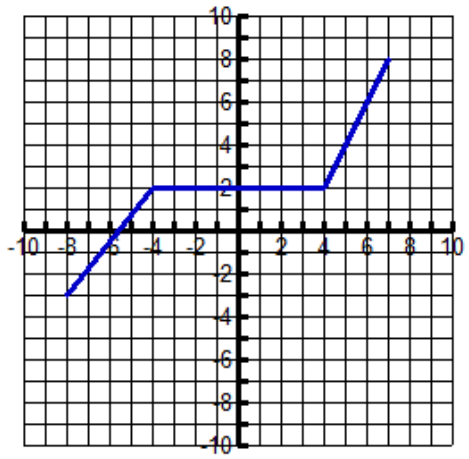
c. Sketch a graph of the piecewise function.



32. What is the inverse of $y = -3x + 9$? Graph both the original function and its inverse.



33. Write the piecewise function that is demonstrated by the graph.



34. Find the inverse of a function given the function and a table of values.

a. $f(x) = 2x + 4$ $\quad f^{-1}(x) =$

x	$f(x)$
-2	
-1	
0	
1	
2	

x	$f^{-1}(x)$

b. $f(x) = \frac{2}{3}x + 6$ $\quad f^{-1}(x) =$

x	$f(x)$
-6	
-3	
0	
3	
6	

x	$f^{-1}(x)$

c. $f(x) = x^2 + 3$ with $x \geq 0$

x	$f(x)$

x	$f^{-1}(x)$

Chapter 5

Perform the indicated operation.

1. $(2x^4+9x-9)-(x^4-2x^2+4x-8)$

2. $(x^4-x^3+x^2-x+1)+(x+x^4-1-x^2)$

3. $(x^2-6y)(x^2+6y)$

4. $(w+4)(w^2+6w-11)$

5. $(x-2)^3$

6. (x +4) (x – 6) (x – 5)

7. $(8x - 2)(8x + 2)$

8. $(3x + 4)^2$

9. The storage space in a moving truck is shaped like a rectangular prism. It has a total volume of 16 cubic meters. The height and width are both 2 meters less than the depth. What are the dimensions of the storage space?

Factor the following expressions completely.

10. $x^3 + 5x^2 + 4x + 20$

11. $2x^3 - 54$

12. $5x^3 - 20x^2 - 25x$

13. $x^4 - 2x^2 - 24$

14. $8y^4 - 18$

15. $x^3 - 4x^2 + 4x$

16. $x^3 + 3x^2 + x + 3$

17. $8x^3 - 27$

18. $x^4 + 7x^2 + 10$

19. $m^3 + 6m^2 - 4m - 24$

Solve by factoring. Find all real solutions.

20. $4x^3 + 16x^2 - 9x - 36 = 0$

21. $x^3 - 27 = 0$

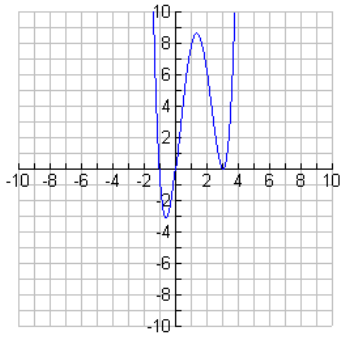
22. $x^3 + 5x^2 - x - 5 = 0$

23. $3x^4 + 30x^2 - 72 = 0$

24. $s^4 + 5s^2 - 6 = 0$

25. A rectangular shipping container has a volume of 96 m^3. The container is 8m taller than it is wide and its length is 2m less than it is wide. What are the dimensions of the container?

27. Find the zeros of the following function.



28. Factor completely.

Factoring cubes: $(a - b)(a^2 + ab + b^2)$

a. $x^3 + 5x^2 + 4x + 20$

b. $x^3 - 27$

c. $4x^3 - 10x^2 - 24x$

d. $x^4 - 6x^2 + 8$

29. Divide using long division. SHOW WORK! $(x^3 - 4x^2 - 2x + 3) \div (x + 1)$

30. Find **all** zeros of the polynomial function. $f(x) = x^4 + 4x^3 + 7x^2 + 16x + 12$

31. Write a polynomial function in STANDARD FORM of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros. Given zeros: 2, 4i

32. If $2 + 3i$ is a solution to a polynomial, then what else is a solution to the same polynomial?
Write an equation with a leading coefficient of 1 with the 2 + 3i and the other zero that needs to be in the same polynomial.

33. Identify the number of solutions or zeros. $y = 2x^5 - 7x^3 - x^2 + x - 4$

34. Write a polynomial function in STANDARD FORM of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros. -1, 3i

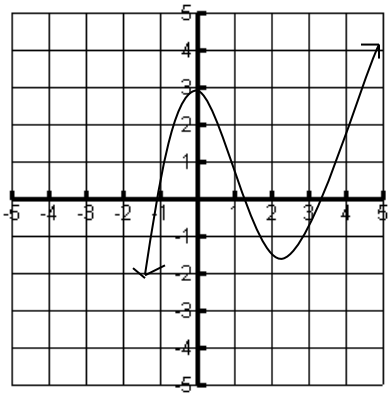
35. Find all zeros of the polynomial function. $f(x) = x^3 - 8x^2 + 11x + 20$

36. Find **all** zeros of the polynomial function. $f(x) = x^4 + x^3 + 2x^2 + 4x - 8$

37. Write a polynomial function in STANDARD FORM of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros. -4, (3 + i), (3 – i)

38. What is true about the polynomial function whose graph is shown?

(Circle all that apply)



a) $f(x) \to -\infty \; as \; x \to +\infty$

b) $f(x) \to +\infty \; as \; x \to -\infty$

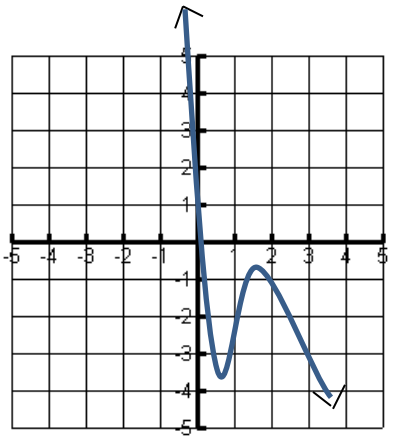
c) Leading coefficient is positive

d) Degree is even

e) Leading coefficient is negative

f) Degree is odd

39. Which equation is the graph of the polynomial function shown?



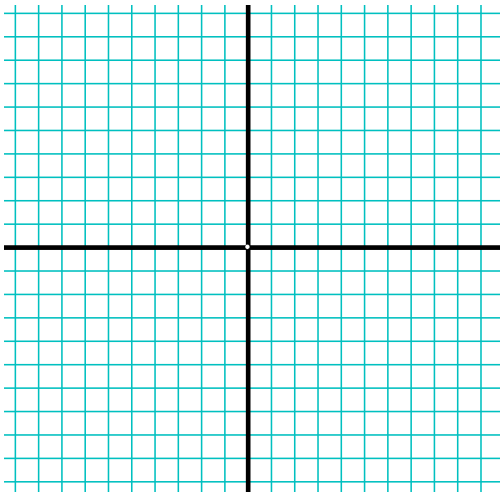
a) $f(x) = 3x^4 - x^2 + 2$

b) $f(x) = 3x^3 - x + 7$

c) $f(x) = -2x^4 + x^2 - 1$

d) $f(x) = -2x^3 + x^2 + 2$

40. Graph the polynomial function using a table of values. Use x = -3,-2,-1,0,1,2, 3.



$f(x) = x^3 - 5x$

X	Y

Factor the polynomial completely using any method.

41. $4x^3 + 16x^2 - 9x - 36$

42. $3x^3 - 81$

Find the *real-number* solutions of the equation.

43. $3n^5 + 15n = 18n^3$

44. A wooden board is shaped like a rectangular prism. It has a total volume of 324 cubic inches. The width is 3 inches less than the height and the length is 12 inches longer than the height. What are the dimensions of the board?

What are the dimensions? _____

45. Factor completely.

a. $m^3 + 6m^2 - 4m - 24$

b. $x^4 + 7x^2 + 10$

_____ _____

46. Divide the polynomials using long division.

$8x^4 + 2x^2 - 12x + 9 \quad \div \quad x^2 + x - 3$

47. Solve by factoring.

$4x^3 + 16x^2 - 9x - 36 = 0$

48. Find *all* zeros of the polynomial functions.

$f(x) = 2x^4 - 2x^3 - 3x^2 - 7x + 10$

49. Write a polynomial function of least degree that has rational coefficients, a leading coefficient of 1, and the zeros are -1 and $3 + 2i$.

_____

50. Graph the function using your graphing calculator. (CALCULATOR!)

a) Find all the local maximums and minimums. Round to the nearest hundredth.
b) Estimate the zeros
c) Justify the amount of zeros you have on the graph. Explain why you can or cannot view all the zeros on the calculator.

$$f(x) = x^6 - 2x^5 + 3x^4 - 10x^3 - 6x^2 - 8x - 8$$

_____

51. Divide the polynomials using long and synthetic division.

$(3x^4 + 2x^2 - 12x + 9) \quad \div \quad (x - 2)$

_____

52. Write a cubic function in factored form whose graph passes *through* the points below. Show all work.

(2, 0), (3, 0), (0, -2), (-3, 0)

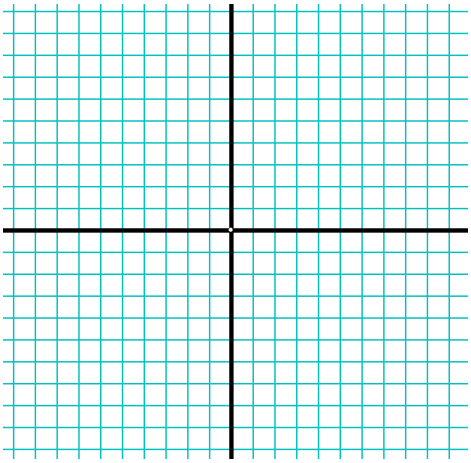
_____

53. Write a polynomial function of least degree that has rational coefficients, a leading coefficient of 1, and the zeros are 6 and $1 - \sqrt{3}$.

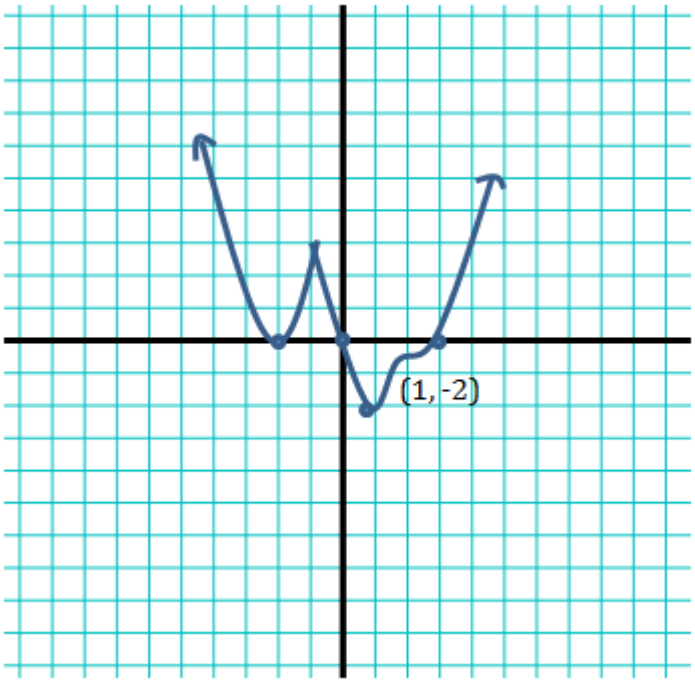
_____

54. Sketch the function.

$$f(x) = -(x-4)^2(x+3)^3(x-1)(x+7)$$



55. Write the equation in factored form whose graph is shown below.



Show work:

Equation: _____Domain: _____Range: _____

56. Graph the function using your graphing calculator. Round to the nearest hundredth. (CALCULATOR)

$f(x) = 3x^3 + 10x^2 - 23x + 7$

a) Find x-intercepts: _____

b) Find the coordinate point of the local maximum(s): _____

c) Find the coordinate point of the local minimum(s)points: _____

57. List the potential zeros for the function: $\boldsymbol{f(x) = 2x^3 + 7x - 12}$

58.

Total Car Sales: The following data represents the total car sales *S* (used plus new car sales) in thousands of cars in the United States for the years 1990-1998, where *x* = 1 represents 1990, *x* = 2 represents 1991, and so on.

Year, x	Total Car Sales, S
1990, **1**	46,830
1991, **2**	45,465
1992, **3**	45,163
1993, **4**	46,575
1994, **5**	49,132
1995, **6**	50,353
1996, **7**	49,355
1997, **8**	48,542
1998, **9**	46,900

a) Find the polynomial function of best fit. Round to two decimal places.

S(x) = _____

b) Use the function to find the total car sales in 2000.

_____

59. You are making a rectangular box out of a 8 x 12 inch piece of cardboard. The box will be formed by making the cuts shown in the diagram and folding up the sides. You want the box to have the greatest volume possible.

a. How long should you make the cuts?

b. What is the maximum volume?

c. What will the dimensions of the finished box be?

a. _____

b. _____

c. _____

CHAPTER 6 (5.1)

Rewrite the expression using radical notation.

1. $(-2)^{\frac{1}{5}}$ 2. $(2/3)^{\frac{1}{2}}$ 3. $(xy)^{\frac{7}{5}}$ 4. $(-3)^{\frac{2}{3}}$

Evaluate the expression without using a calculator.

5. $(16)^{\frac{3}{4}}$ 6. $\left(\sqrt[3]{-64}\right)^2$ 7. $(-1000)^{\frac{2}{3}}$ 8. $\left(\sqrt{25}\right)^2$

Simplify using exponent properties.

9. $\frac{(c^4)^3}{4} \cdot \frac{12d^{-6}}{(6cd)^{-1}}$ 10. $\left(\frac{2a^{-3}b^2}{6a^2b^4}\right)^{-1}$

11. $\left(\frac{4a^{-2}b^{6}}{8a^{5}b^{10}}\right)^{-2}$

12. $\frac{(x^{4})^{3}y}{4} \cdot \frac{6x^{-5}}{2(yx)^{-1}}$

Simplify the expressions and write solution in scientific notation.

13. $\frac{52\ x\ 10^{-3}}{2\ x\ 10^{-5}}$

14. $\frac{46\ x\ 10^{-3}}{2\ x\ 10^{-7}}$

15. $(3.0\ x\ 10^{-5})_{x}\,(2.5\ x\ 10^{-3})$

16. $(12.0\ x\ 10^{5})_{x}\,(3\ x\ 10^{-2})$

Evaluate the expression without using a calculator.

17. $(81)^{\frac{3}{4}}$

18. $\left(\sqrt[3]{-27}\right)^2$

19. $(16)^{\frac{-3}{4}}$

20. $\left(\sqrt[3]{-1000}\right)^2$

21. $\sqrt[3]{27} \cdot \sqrt[3]{81}$

22. $\sqrt[4]{32} \cdot \sqrt[4]{24}$

23. $-2(2x+3)^3 = 54$

24. $2x^3 + 20 = 36$

25. $x^5 - 36 = -4$

26. $24 + (x + 6)^4 = 25$

Simplify the expression. Assume all variables are positive. No rational exponents or radicals can be left in the denominator.

27. $\sqrt{\frac{32x^5}{27x^3}}$

28. $\left(\frac{6f^{11/3}}{3f^{7/3}}\right)$

29. $\sqrt[3]{(3x^3)^2(3y^2)^5}$

30. $\sqrt[3]{(4xy^2)^2(2x^3y^2)^5}$

31. $\left(\left(\frac{2x^{11/6}}{x^{4/3}}\right)^2\right)^3$

32. $\sqrt{\frac{16y^8}{3y^3}}$

Perform the indicated operation.

33. $2\sqrt[4]{xy} - 5\sqrt[4]{xy}$

34. $3\sqrt[5]{64} - 8\sqrt[5]{2}$

35. $xy\sqrt[4]{48xy^4} - 5x\sqrt[4]{y^8 3x}$

36. $y^3\sqrt[5]{32x^4} - 7\sqrt[5]{x^4y^{15}}$

37. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

38. $\sqrt[5]{\frac{x^{10}}{y^5}}$

39. $\frac{6xy^{3/4}}{3x^{1/2}y^{1/2}}$

40. $\sqrt{9w^5} - w\sqrt{w^3}$

41. $\frac{3}{\sqrt[4]{144}}$

Calculator Section Problems

Evaluate the expression using a calculator. Round the result to the nearest hundredth where appropriate.

1. $(-23)^{-2/5}$

2. $\sqrt[5]{-116}$

3. $(-13)^{-2/5}$

4. $\sqrt[5]{-146}$

Solve the equation. Round the result to two decimal places when appropriate. Show all steps algebraically.

5. $x^4 + 9 = 36$

6. $12 - (2x + 3)^3 = 54$

Solve the equation. Round the result to the nearest hundredth where appropriate. Show all steps algebraically.

7. $x^3 + 3 = 67$

8. $4 + (3x + 1)^4 = 20$

Rewrite the expression using radical notation. (Non-Calculator Problems)

9. $(-3)^{\frac{3}{7}}$

10. $(-13)^{\frac{4}{5}}$

_____

_____

Evaluate the expression without using a calculator.

11. $(16)^{\frac{5}{4}}$

12. $\left(\sqrt[3]{-64}\right)^2$

_____

_____

13. $\left(\sqrt[3]{-27}\right)^2$

14. $(81)^{\frac{3}{4}}$

_____

_____

Simplify the expression. Assume all variables are positive.

15. $\left(\frac{5^{2/3}.y^{9/6}}{5^{5/12}.y^{2/3}}\right)$

16. $\sqrt[4]{(3x^2yz)^4\,(5zx^3)^2}$

Simplify the expression. Assume all variables are positive.

17. $\sqrt[4]{(2x^2y)^3(2xy^3)^2}$

18. $\left(\frac{7^{1/6}.x^{7/6}}{7^{1/12}.x^{2/3}}\right)$

Perform the indicated operation. Assume all variables are positive.

19. $2x\sqrt[4]{x^4yz^5} + \sqrt[4]{x^8yz^5}$

20. $5x\sqrt[3]{x^4yz^5} + \sqrt[3]{x^7yz^5}$

Let $\boldsymbol{f(x) = 5 - x^2,\ g(x) = 3x + 6,\ h(x) = \sqrt{2x}, and\ r(x) = \frac{3x+1}{7}}$. Perform the indicated operation. **Then state the domain.**

21. $\frac{f(x)}{g(x)}$

22. $r(r(x))$

Let $\boldsymbol{f(x) = 3 - x^2,\ g(x) = 2x - 1,\ h(x) = \sqrt{x + 1}, and\ r(x) = \frac{x+3}{2}}$.
Perform the indicated operation. **Then state the domain**.

23. $g(x) - f(x)$

24. $f\big(h(x)\big)$

25. $\frac{f(x)}{g(x)}$

26. $r(r(x))$

Let $\boldsymbol{f(x) = 5 - x^2,\ g(x) = 3x + 6,\ h(x) = \sqrt{2x}, and\ r(x) = \frac{3x+1}{7}}$.

Perform the indicated operation. **Then state the domain.**

27. $g(x) - f(x)$

28. $f\big(h(x)\big)$

Solve the equations. Check for extraneous solutions.

29. $(2x + 3)^{3/2} = 27$

30. $\sqrt{x + 10} = x - 2$

31. $(3x^2 - 2)^{1/2} + 4 = 9$

32. $\sqrt{3x + 7} = x + 1$

33. Solve the equation using algebra and check for extraneous solutions. Show all work algebraically.

$\sqrt{2x+3}+2=\sqrt{6x+7}$

Calculator Section. Round all answers to two decimal places if necessary.

34. Solve the equation by graphing on the calculator. $\sqrt[3]{{}^{2}/_{3}\,x-1}=\sqrt{x}-5$

35. Solve the equation by graphing on the calculator. (No algebra required)

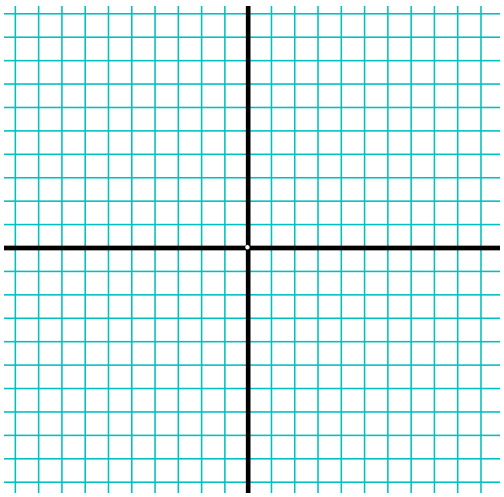
$\sqrt[3]{{}^{7}/_{8}\,x-9}=-\sqrt{x}+3$

Chapter 7.1-7.3

Non-Calculator Section

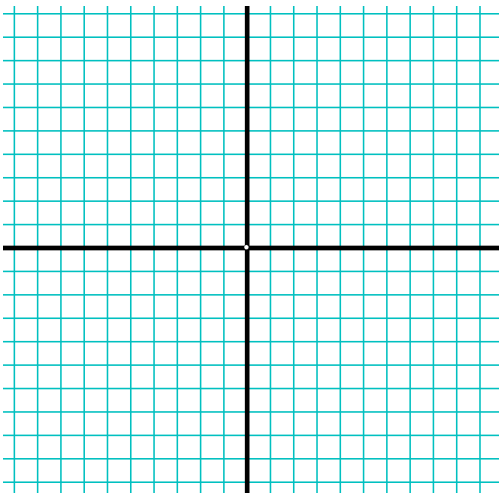
Graph the function. State the domain and range.
Plot and label three points and draw in asymptotes. Show work for how you found the three points.

1. $y = \left(\frac{2}{3}\right)^{x+1} - 5$



Domain: _____
Range: _____

2. $y = -(4)^{x-2} + 1$



Domain: _____
Range: _____

Classify the following as exponential growth or decay, then state the domain and range.

3. $y = 2\left(\frac{1}{3}\right)^{x-1} + 4$

Growth or decay: _____
Domain: _____
Range: _____

4. $y = 3\left(\frac{5}{4}\right)^{x} - 1$

Growth or decay: _____
Domain: _____
Range: _____

5. $y = \frac{1}{3}e^{2x}$

G/D: _____
Why?

6. $y = 5e^{-x}$

G/D: _____
Why?

Simplify the expression.

7. $\frac{e^{-4}(9e^2)^2}{3}$

8. $\sqrt[3]{16e^{21x}}$

9. $\frac{1}{3}e^{-2}(6e^3)^2$

10. $\sqrt[3]{24e^{13x}}$

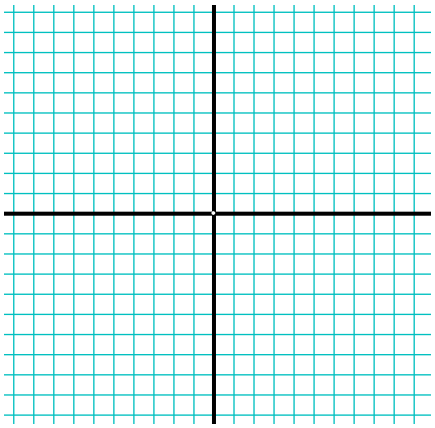
11. The equation represents the value of a car over time: $y = 5{,}000(.624)^t$ where *t* is the number of year since the car was purchased.

a) State the original amount of the car: _____
b) State the decay factor: _____
c) State the percent decrease: _____

Calculator Section

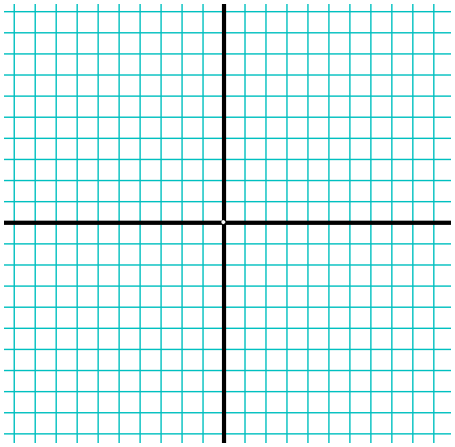
Graph the function. State the domain and range.
Sketch and draw in asymptotes.

12. $y = 2e^{-3x} + 4$



Growth or Decay?_____
Domain: _____
Range: _____

13. $y = 3e^{-x} - 1$



Growth or Decay?: _____
Domain: _____
Range: _____

Evaluate the expressions. Round to three decimal places.

14. $e^{\frac{\sqrt{2}}{3}}$

15. e^{3}

16. In 6 years, you want to have $2,000 in your savings account. Find the amount you should deposit if the account pays 3% annual interest, compounded monthly. ***Show all your steps algebraically.***

17. You are choosing between three banks to invest your money. You call around to several local banks to find out how they compound interest. The money will be in the bank for 5 years. You will be investing $3000.

Bank A compounds it semiannually at a rate of 3.1%
Bank B compounds continuously at a rate of 3%
Bank C compounds quarterly at a rate of 2.95%

How much <u>interest</u> would you earn from each bank?

18. The value of a car can be modeled by the equation $y = 24{,}000(.745)^{t}$

Where t is the number of year since the car was purchased. Round to the nearest hundredth, if necessary.

- What is the original price of the car? _____
- What is the decay factor? _____
- What is the percent decrease? _____
- After how many years will the car be worth $15,000? _____
- What is the value of the car after 20 years?
- Is this a reasonable value? *Explain.*

19. You buy a new iPad for $1200 and are able to sell it 5 years later for $700. Assume that the resale value of the iPad players decays exponentially with time. Write an equation giving the iPad players resale value V (in dollars) as a function of the time *t* (in years) since you bought it. Round to the nearest hundredth, if necessary.

20. You were just a competitor on the Road Rules Challenge and won $25,000. Now that you are home you have decided to put in the bank and save for college. You call around to several local banks to find out how they compound interest. The money will be in the bank until you leave for college in 3 years.
Bank A compounds it semiannually at a rate of 2.2%
Bank B compounds continuously at a rate of 2.1%
Bank C compounds quarterly at a rate of 2.15%

Bank A

Bank B

Bank C

Which bank would you go with? Explain your reasoning.

21. In 4 years, you want to have $16,000 in your savings account. Find the amount you should deposit if the account pays 3% annual interest, compounded monthly. ***Show all your steps algebraically.*** Round to the nearest hundredth if necessary.

22. A new all-terrain vehicle (ATV) costs $9,000. The value of the ATV decreases by 7.5% each year. Round to the
nearest hundredth if necessary.

a) Write a model for the value of the ATV, *y (in dollars)* after *t* years.
b) Estimate the value after 7 years.