Precalc 41 Midterm review Name_____

Chapter 1

1. State the domain using interval notation.

a. $h(x) = x^2 - 8x + 1$

b. $g(x) = \sqrt{-3x - 2}$

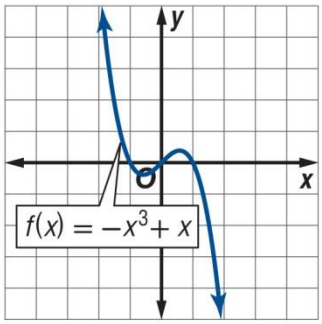
c. $f(x) = \dfrac{12x}{x^2 - x - 12}$

2. Find f(-2), f(2x+1), f(a), and $\dfrac{f(a+h) - f(a)}{h}$ for the function $f(x) = 2x^2 - 7x$.

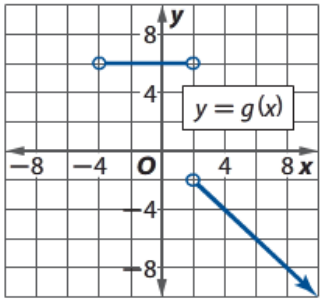
a. f(-2)
b. f(2x+1)
c. f(a)
d. f(a + h)
e. $\dfrac{f(a+h) - f(a)}{h}$

3. Using the functions below, find the following domain, range and interval(s):

a.



b.



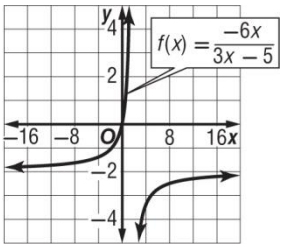
4. Test each of the following functions for symmetry with respect to the *x*-axis, *y*-axis, and origin. Determine whether the function is even, odd, or neither.

a. $y = x^4 - 8x^2$ b. $y = 2x + 8$ c. $y = x^3$ d. $xy = 12$

5. Find the average rate of change of each function on the given interval. $f(x) = x^4 + 2x^3 - x - 1$; [−3, −2]

6. Describe the end behavior of each function using limit notation.

a.

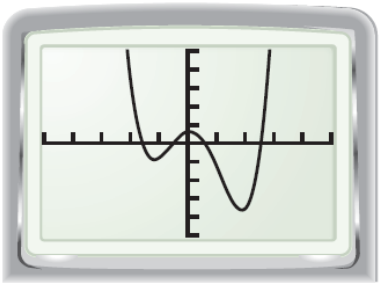


b. $h(x) = -4x^6 - 5x^2 + 2$

c. $f(x) = 2x(x + 3)^2(x - 5)$

7. Approximate the following from the graph.
(The interval for both the x and y axis are by one)

a. Domain
b. Range
c. Roots (x-intercept(s))
d. Y-intercept
e. Local maximum, if any.
f. Local minimum, if any.



8. Given $f(x) = \dfrac{2x+6}{x+1}$, answer the following questions:

a. Is the point $(-2,-2)$ on the graph of f?
b. If $x = -7$, what is $f(x)$?
c. If $f(x) = 4$, what is the value(s) of x?
d. What is the domain of $f(x)$?

9. Write the function whose graph is the graph of $y = x^3$, but is stretched vertically by a factor of 2, shifted to the left 1 unit, and shifted up 3 units.

10. Prove the following functions are inverses. $f(x) = \dfrac{1}{7}x - \dfrac{3}{7}$ and g(x) = 7x + 3

11. Determine the equation for the inverse of the functions. Circle the functions that are (one-to-one).

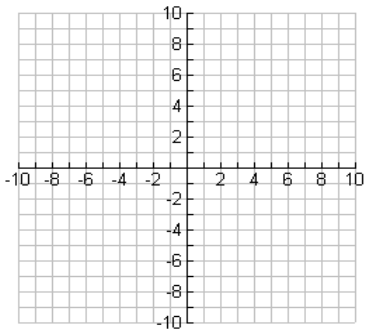
a. $g(x) = 2x - 6$

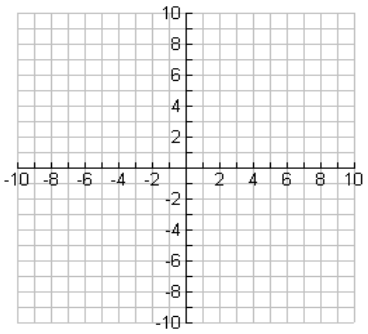
b. $f(x) = (x-1)^3 + 1$

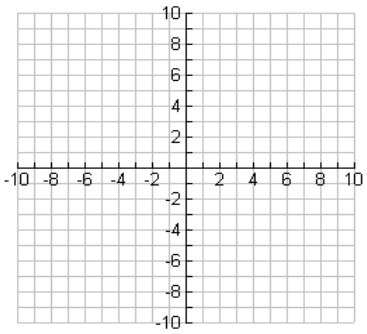
c. $f(x) = (x - 3)^2 - 2$

12. Graph the following functions.

a. $f(x)=\dfrac{-3}{x+4}+1$ b. $g(x)=\sqrt{x-5}+2$ c. h(x) = $h(x)=-2|x|-3$







Chapter 2 Midterm Review

1. State the domain of the function.

$$h(x) = -\frac{1}{2}\sqrt[10]{2x-6}+1$$

2. Solve the equation. Write your answer in interval notation.

$$(3-2x)^{3/5} \geq 8$$

3. Solve the equation: $\sqrt{3x+4}-\sqrt{2x+1}=1$

4. How many positive and negative real zeros does the function f(x) have?
$\boldsymbol{f(x)=3x^5-2x^4+8x^3-2x^2-10x-6}$.

5. For the function $f(x)=x^4-x^3-6x^2$
   a. Determine the end behavior of the graph.

   b. Determine the zeros and state the multiplicity of any repeated zeros

   c. Use this information to sketch a graph of the function.

6. Use long division to divide $(2x^4 + 5x^3 - 5x^2 + 3x - 18) \div (2x - 1)$.

7. Use synthetic division to divide $(x^3 - 8x^2 - 15) \div (x - 1)$.

8. Determine the remainder when $(4x^4 - 8x^3 + 12x^2 - 6x + 12)$ is divided by $(x + 1)$.

9. Write the function $h(x) = x^4 - 3x^3 + 6x^2 - 12x + 8$

a) Find the possible rational zeros of *h(x).*

b) Find all the zeros of *h(x)*. (4 pts)

c) Write h(x) as the product of linear and irreducible quadratic factors.

d) Write *h(x)* as the product of linear factors.

10. Write a polynomial function in standard form with a leading coefficient of one with the following zeros: x = -1 and x = (2 + i)

11. Given the function $f(x) = 2x^4 - 2x^3 - 3x^2 - 7x + 10$

a) How many positive real zeros are possible? _____

b) How many negative real zeros are possible? _____

c) What are the possible rational zeros? _____

d) Is it possible that there are no real zeros? _____

e) Are there any real zeros greater than 2? _____

12. Solve the inequality: $\frac{(x-3)(x+2)}{(x-1)} \leq 0$

13. For each of the following:

i. Determine the domain of the function.
ii. Determine the *x* and *y* intercepts.
iii. Determine the vertical asymptote(s).
iv. State, if any, any horizontal or oblique asymptote(s).

a. $f(x) = \frac{4x}{x+1}$

b. $f(x) = \frac{x^2+5x+6}{x+3}$

14. Determine the equations of the asymptotes and/or holes for the following functions:

a. $f(x)=\dfrac{2x^3+7x^2-4}{x^2+2x-3}$

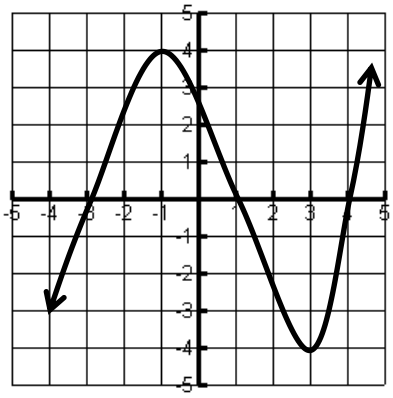
b. $g(x)=\dfrac{x+2}{x^2+2x-3}$

c. $k(x)=\dfrac{2x^2+5x-1}{5x^2}$

d. $h(x)=\dfrac{x^2+5x-6}{2x^2-5x+3}$

15. Approximate the following from the graph.
(The interval for both the x and y axis are by one)

a. Domain
b. Range
c. X-intercepts
d. Y-intercepts
e. Local maximum:
f. Local minimum:
g. Absolute Maximum:
h. Absolute Minimum:
i. Increasing:
j. Decreasing:
k. End behavior:



16. Simplify the expressions.

a. $\dfrac{x^2-16}{9-x}\cdot\dfrac{x^2+x-90}{x^2+14x+40}$

b. $\dfrac{n+3}{n+2}\div\dfrac{n^2+2n-3}{(n-1)^2}$

c. $\dfrac{7x}{x+1}+\dfrac{8}{x-7}$

d. $\dfrac{4}{x+1}-\dfrac{2}{x+2}$

17. Solve the equation.

a. $\dfrac{1}{x}+\dfrac{3x+12}{x^2-5x}=\dfrac{7x-56}{x^2-5x}$

b. $\dfrac{1}{x-2}+\dfrac{1}{x^2-7x+10}=\dfrac{6}{x-2}$

c. $\dfrac{x+5}{x^2-2x}-1=\dfrac{1}{x^2-2x}$

d. $\dfrac{5}{x^3+5x^2}=\dfrac{5}{x+5}+\dfrac{1}{x^2}$

CALCULATOR:

18. Solve: $x^4 + 5x^2 - 3x = 50$ (round to nearest Hundredth)

19. Determine the coefficient of the sixth term in the expansion of $(x - 4)^7$.

20. Expand: $(2x + 3)^4$.

21. *The following data represents the results of an experiment involving the growth of bacteria.

Time (x) (in minutes)	1	3	5	7	9	11
Number of Bacteria (y)	2	25	81	175	310	497

a. Write a power regression equation for this set of data, rounding all values to *three decimal places.*

b. Using this equation predict the bacteria's growth, to the nearest integer, after 15 minutes.

Chapter 3 Midterm Review

1. Re-write the following exponential statements as logarithmic statements.

a. $t = 4^x$ b. $w = e^4$

l. Re-write the following logarithmic statements as exponential statements.

a. $b = \log_w j$ b. $v = \log \pi$

m. For each of the following functions describe its domain, range, intercepts, asymptotes, end behavior, and whether the function is increasing or decreasing.

a. $f(x) = 6^{x-2} + 4$ b. $g(x) = \log_4(x + 1) - 2$ c. $g(x) = -5\log(x - 4)$

4. What is final balance of an $800 invested in an account that pays 12% compounded quarterly for 7 years?

5. An initial investment of $1,300 became $2,500 after 10 years of earning at a continuously compounding rate. What was the rate of return?

6. How long must $20,000 be invested at a 9% rate compounding semi-annually in order to become $40,000?

7. When infected with Virus D a population of ants falls to 40% of its initial population after 18 hours. How long until there is only 20% left?

8. Evaluate each expression:

a. $\frac{\log_5 625}{\log_5 125}$ b. $\log_8 \frac{1}{64}$ c. log. 001 d. $8\log_4 \sqrt[3]{64}$

9. Expand each expression:

a. $\log_2 \frac{3g^2p^{-3}}{m^5g^{.5}}$ b. $\log b^{-3}\, x^2 w^{-5}$

10. Condense each expression:

a. $\ln 12 + \ln b - 4\ln d$

b. $\frac{1}{2}\log_5(b-2a) - \frac{1}{3}\log_5(2c-d)$

11. Solve each expression:

a. $\log_3(x^3+25) = \log_3 52$

b. $10^{5x-6} = 100^{2x+1}$

c. $\log_7(5x-7) = 3$

d. $\log_9 2x = \log_9 196 - \log_9 2x$

e. $325 + 6\log_4 x = 349$

f. $7e^{3x-2} - 10 = 39$

g. $6^{4x+2} = 15^{x+1}$

h. $e^{2x} + 10e^x - 24 = 0$

12. Match the equation with the type of regression that it could have produced.

a. Power Regression

b. Logarithmic Regression

c. Exponential Regression

d. Logistic Regression

1) $y = 2.3x^{.567}$

2) $\hat{y} = 8.22 * .78^x$

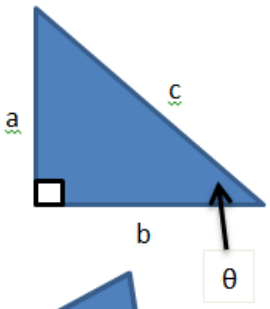
3) $\hat{y} = \frac{21.4}{1+3.12e^{-.62x}}$

4) $\hat{y} = 2.2\ln(x) - 3.24$

Midterm Review – Chapter 4: Sections 4-1 through 4-4
Section 4-1 – Right Angle Trigonometry
Key Terms: Sine, Cosine, Tangent, Secant, Cosecant, Cotangent, Angle of Elevation, Angle of Depression, Legs of Right Triangle, Hypotenuse of a Right Triangle,

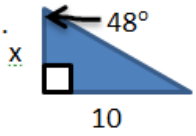
Practice Problems

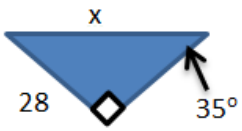
1. Define the terms sine, cosine, tangent, secant, cosecant, and cotangent using the right triangle



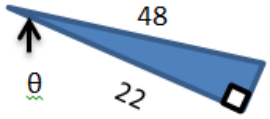
2. Given that θ is acute and $\cos\theta = \frac{6}{7}$, find the exact values of the other 5 functions.

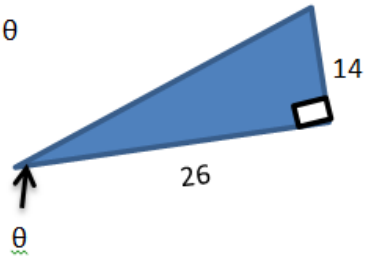
In 3 and 4, find the value of x.

3.


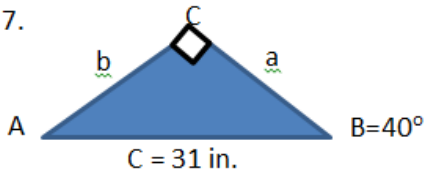
4.


In 5 and 6, find the measure of θ

5.


6.


In Number 7, solve the triangle

7.


Section 4-2: Radians and Degrees

Key Terms: Vertex, initial side, terminal side, standard position, positive angle, negative angle, radian, degree, coterminal angles, arc length, area of a sector, linear speed, angular speed, central angle

Practice Problems

1. Convert 45° to radians

2. Covert $\frac{3\pi}{4}$ radians to degrees

3. Find 2 positive and 2 negative angles coterminal to 226°.

In 4 and 5, find the length of the intercepted arc of the circle with given radius and angle measure

4. radius = 4 inches and $\alpha = \frac{\pi}{3}$ radians

5. radius = 9 cm and θ = 150°

6. Find the area of the sector: $\alpha = \frac{2\pi}{5}$ radians and r = 25 inches

7. Find the area of the sector: θ = 210° and radius = 3 ft

8. A wall clock has a diameter of 8.5 inches. The hour hand has a length of 2.4 inches, the minute hand is 3.2 inches, and the second hand is 3.4 inches. Determine the angular speed (radians per hour) and linear speed (inches per hour) for each hand.

6.4 partial fractions

Partial fractions (6.4)(calculator is allowed for these problems)

1. Find the partial fraction decomposition of each rational expression.

a. $\dfrac{x+1}{x^2+5x+6}$

b. $\dfrac{-2x^3+4x^2+22x-32}{x^3+2x^2-8x}$

c. $\dfrac{-x^2-22x-50}{x^3+10x^2+25x}$

d. $\dfrac{3x^4+4x^2+8x+18}{x(x^2+3)^2}$

ANSWERS

Chapter 1 Solutions:

1a. $(-\infty,\infty)$

1b. $\left(-\infty,-\frac{2}{3}\right]$

1c. $(-\infty,-3)U(-3,4)U(4,\infty)$

2a. 22

2b $8x^2 - 6x - 5$

2c. $2a^2 - 7a$

2d. $2a^2 + 4ah + 2h^2 - 7a - 7h$

2e. $4a + 2h - 7$

3a. D: $(-\infty,\infty)$ R: $(-\infty,\infty)$ increasing: (-1/2,1/2) decreasing:$(-\infty, -½)(1/2, \infty)$ constant: N/A

3b. D: $(-4,2)U(2,\infty)$ R: $(-\infty,-2)U[6]$ increasing: N/A decreasing: $(2, \infty)$ constant: (-4,2)

4a. y-axis symmetry, even

4b. no symmetry, neither

4c. origin symmetry, odd

4d. origin symmetry, odd

5. -28

6a. $\lim_{x\to\infty} g(x) = -2$ $\lim_{x\to-\infty} g(x) = -2$

6b. $\lim_{x\to\infty} g(x) = -\infty$ $\lim_{x\to-\infty} g(x) = -\infty$

6c. $\lim_{x\to\infty} g(x) = \infty$ $\lim_{x\to-\infty} g(x) = \infty$

7a. $(-\infty,\infty)$

7b. $[-4,\infty)$

7c. (-1.5,0) , (-.5,0) , (2.5,0) , (.5, 0)

7d. (0, ½)

7e. .5 at x = 0

7f. -1 at x = -1 and -4 at x = 2

8a. yes

8b. 4/3

8c. x = 1

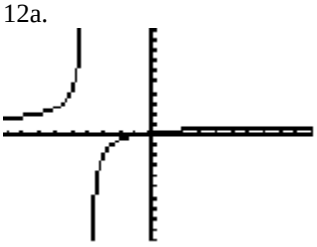
8d. $(-\infty,-1)U(-1,\infty)$

9. $y = 2(x + 1)^3 + 3$

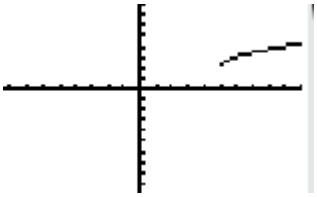
10. Show f(g(x)) = g(f(x)) = x

11a. $g^{-1}(x) = \frac{1}{2}x + 3$ (one-to-one) 11b. $f^{-1}(x) = \sqrt[3]{x-1} + 1$ (one-to-one) 11c. $h^{-1}(x) = \pm\sqrt{x+2} + 3$ (not one-to-one)

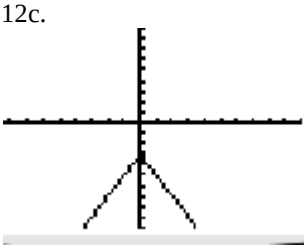
12a.



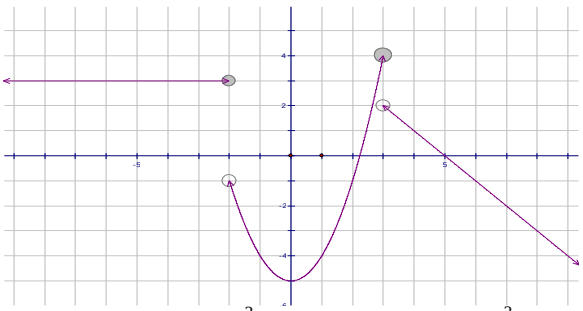
12b.



12c.



13.



14a. $V(x) = x(24 - 2x)^2$ 14b. $V(10) = 160$ in^3 14c. The volume is largest when $x = 4$. The maximum volume is 1024 in^3

15. 34 feet by 68 feet

Chapter 2 Answers

1. $[3, \infty)$
2. $(-\infty, -14.5]$
3. $x = 0, 4$
4. 3 or 1 positive zeros
   2 or 0 negative zeros
5. a. $\lim_{x\to-\infty} f(x) = \infty$
   $\lim_{x\to\infty} f(x) = \infty$
   b. zeros: 0 mult. 2
   3 mult 1
   -2 mult 1
   c. 

6. $x^3 + 3x^2 - 1x + 1 - \frac{17}{2x-1}$
7. $x^2 - 7x - 7 - \frac{22}{x-1}$
8. Remainder: 42
9. a. ±1, ±2, ±4, ±8
   b. 1, 2, ±2i
   c. $(x^2 + 4)(x - 1)(x - 2)$
   d. $(x - 2i)(x + 2i)(x - 1)(x - 2)$
10. $x^3 - 3x^2 + x + 5$
11. a. 2 or 0 zeros
    b. 2 or 0 zeros
    c. $\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{2}, \pm\frac{5}{2}$
    d. yes
    e. no
12. $(-\infty, -2], (1,3]$
13. ai. $(-\infty, -1), (-1, \infty)$
    ii. x-int: x = 0
    y–int: y = 0
    iii. VA: x = -1
    iv. HA: y = 4

    b i. $(-\infty, \infty)$
    ii. X-int: x = -2
    y-int: y = 2
    iii. None
    iv: none: hole at x = -3

14. a. Oblique A: y = 2x + 3 VA: x = -3, x = 1
    b. HA: y = 0 VA: x = -3, x = 1
    c. HA: $y = \frac{2}{5}$ VA: x = 0
    d. HA: y = ½ VA: x = 3/2 hole (1, -7)

15. a: Domain: $(-\infty, \infty)$
    B: Range: $(-\infty, \infty)$
    c. (-3,0) , (1,0), (4, 0)
    d. (0, 2.5)
    e. (-1,4)
    f. (3, -4)
    g. none
    h. none
    i. $(-\infty, -1)$ and $(3, \infty)$
    j. (-1, 3)
    k. $\lim_{x\to-\infty} f(x) = -\infty$
    $\lim_{x\to\infty} f(x) = \infty$

16. a. $-(x - 4)$ b. $\frac{n-1}{n+2}$
    c. $\frac{7x^2-41x+8}{(x+1)(x-7)}$ d. $\frac{2x+6}{(x+1)(x+2)}$

17. a. x = 21
    b. $x = \frac{26}{5}$
    c. x = 4, -1
    d. $x = -\frac{1}{4}$

Calculator

18. x = -2.14, x = 2.34
19. -21504
20. $16x^4 + 96x^3 + 21x^2 + 216x + 81$
21. a. $y = 2.001 \cdot x^{2.298}$
    b. y = 1009

Chapter 3 Answers

a.a.$\log_4 t = x$ $b. \ln w = 4$

b. a. $j = w^b$ b. $10^v = \pi$

c. For each of the following functions describe its domain, range, intercepts, asymptotes, end behavior, and whether the function is increasing or decreasing.

a. b. c.

	$f(x) = 6^{x-2} + 4$	$g(x) = \log_4(x+1) - 2$	$g(x) = -5\log(x-4)$
Domain	$(-\infty, \infty)$	$(-1, \infty)$	$(4, \infty)$
Range	$(4, \infty)$	$(-\infty, \infty)$	$(-\infty, \infty)$
Intercepts	(0,4 1/36)	(15,0)(0,-2)	(5,0)
Asymptotes	Y=4	X=-1	X=4
End Behavior	$\lim_{x\to-\infty} = 0$ $\lim =_{x\to\infty} \infty$	$\lim_{x\to-1} = -\infty$ $\lim =_{x\to\infty} \infty$	$\lim_{x\to4} = \infty$ $\lim =_{x\to\infty} -\infty$
Increasing or Decreasing	Increasing	Increasing	Decreasing

d. $1,830.34

e. 6.54%

f. 7.8 years

g. 31.62 hours

h.a. $\frac{4}{3}$ b. -2 c. -3 d. 8

i. a. $\log_2 3 + 1.5\log_2 g - 3\log_2 p - 5\log_2 m$ b. $-3\log b + 2\log x - 5\log w$

j. a. $\ln \frac{12b}{a^4}$ b. $\log_5 \frac{(b-2a)^{1/2}}{(2c-d)^{1/3}}$ or $\frac{(2c-d)^{2/3}(b-2a)^{1/2}}{(2c-d)}$

k. Solve each expression:

a. $x = 3$ b. $x = 8$ c. $x = 70$ d. $x = \pm 7$ e. $x = 256$ f. $x = 1.315$

g. $x = -.1963$ h. x=.6931

l. a. & 1), b. & 4), c. & 2), d. & 3)

Chapter 4 Answers:

<u>Section 4-1</u>

1. $\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$, $\tan\theta = \frac{a}{b}$, $\csc\theta = \frac{c}{a}$, $\sec\theta = \frac{c}{b}$, $\cot\theta = \frac{b}{a}$

2. $\sin\theta = \frac{\sqrt{13}}{7}$, $\cos\theta = \frac{6}{7}$, $\tan\theta = \frac{\sqrt{13}}{6}$, $\csc\theta = \frac{7}{\sqrt{13}}$, $\sec\theta = \frac{7}{6}$, $\cot\theta = \frac{6}{\sqrt{13}}$

3. x = 9.00 4. x = 48.81 5. $\theta = 62.7^o$ 6. $\theta = 28.3^o$ 7. $A = 50^o$ a = 23.74 b = 19.93

<u>Section 4-2</u>

1. $\frac{\pi}{4}$ rad 2. 135^o 3. $586^o, 946^o, -134^o, -494^o$ 4. $\frac{4\pi}{3}$ inches 5. $\frac{15\pi}{2}$ cm 6. 125π inches^2 7. $\frac{21\pi}{4}$ feet^2

8. a) $\frac{\pi}{6}$, 2π, 120π rad/hr b) 1.26, 20.1, 1281.8 inches/hr

6.4 partial fractions

1a. $\frac{2}{x+3} - \frac{1}{x+2}$

1b. $-2 + \frac{4}{x} + \frac{1}{x-2} + \frac{3}{x+4}$

1c. $\frac{-2}{x} + \frac{1}{x+5} - \frac{7}{(x+5)^2}$

1d. $\frac{2}{x} + \frac{x}{x^2+3} + \frac{-11x+8}{(x^2+3)^2}$