Functions

1. 

\[ f(2) = \] 

\[ f(x) = -2 \] 

\[ f(-3) = \] 

\[ f(x) = 3 \] 

\[ f(0) = \] 

\[ \text{Domain: } \] 

\[ \text{Range: } \] 

Increasing: 

Decreasing: 

Constant: 

2. For a-f, transform each graph.

a) \( f(x) + 2 \) 

b) \( f(x - 3) \) 

c) \( f(x) - 1 \) 

d) \(-f(x)\) 

e) \( f(x + 1) - 3 \) 

f) \(-f(x) + 2 \) 

g) State the domain: 

h) State the range: 

i) Determine: \( f(3) \) 

\( f(-5) \) 

\( f(x) = 1 \) 

3. Given \( f(x) = 2x^2 + 11x - 11 \), find:

a) \( f(-3) \) 

b) \( f(4) \) 

c) \( f(0) \) 

d) \( f(5) \) 

e) \( f(x) = -25 \) 

f) \( f(x) = -23 \)
Imaginary Numbers

Perform the indicated operation, and write in proper complex form.

1. \((3 + 2i)(4 - 5i)\)  
2. \((2 - 6i)^2\)  
3. \((3 + 2i) + (4 - 5i)\)  
4. \((11 + 8i) - (9 - 5i)\)  
5. \((11 + 8i) - (9 - 5i)\)  
6. \((3 + 2i) + (4 - 5i)\)

Polynomials

Perform the indicated operation.

1. \((2x^4 + 9x - 9) - (x^4 - 2x^2 + 4x - 8)\)  
2. \((x^4 - x^3 + x^2 - x + 1) + (x + x^4 - 1 - x^2)\)  
3. \((x^2 - 6y)(x^2 + 6y)\)  
4. \((w + 4)(w^2 + 6w - 11)\)  
5. \((x - 2)^3\)  
6. \((x + 4)(x - 6)(x - 5)\)  
7. \((8x - 2)(8x + 2)\)  
8. \((3x + 4)^2\)

Factor the following expressions completely.

1. \(x^3 + 5x^2 + 4x + 20\)  
2. \(2x^3 - 54\)  
3. \(5x^3 - 20x^2 - 25x\)  
4. \(x^4 - 2x^2 - 24\)  
5. \(8y^4 - 18\)  
6. \(x^3 - 4x^2 + 4x\)  
7. \(x^3 + 3x^2 + x + 3\)  
8. \(8x^3 - 27\)  
9. \(x^4 + 7x^2 + 10\)  
10. \(m^3 + 6m^2 - 4m - 24\)  
11. \(4x^3 - 10x^2 - 24x\)  
12. \(x^4 - 6x^2 + 8\)  
13. \(4x^3 + 16x^2 - 9x - 36\)  
14. \(3x^3 - 81\)  
15. \(m^3 + 6m^2 - 4m - 24\)  
16. \(x^4 + 9x^2 - 10\)

Solve. Find all real solutions.

1. \(4x^3 + 16x^2 - 9x - 36 = 0\)  
2. \(x^3 - 27 = 0\)  
3. \(x^3 + 5x^2 - x - 5 = 0\)  
4. \(3x^4 + 30x^2 - 72 = 0\)  
5. \(s^4 + 5s^2 - 6 = 0\)  
6. \(2x^4 + 10x^3 - 2x^2 - 10x = 0\)  
7. \(-3x^3 - 15x^2 + 3x + 15 = 0\)  
8. \(3n^5 + 15n = 18n^3\)  
9. \(3n^5 + 15n = 18n^3\)
27. Find the zeros of the following function.

Divide using synthetic division. SHOW WORK!

1. \((x^3 - 4x^2 - 2x + 3) \div (x + 1)\)

2. \((3x^4 + 2x^2 - 12x + 9) \div (x - 2)\)

Graphing Polynomials – identify degree, x-intercepts, y intercept, and end behavior.

Find all zeros of the polynomial function and then graph.

1. \(f(x) = x^4 + 4x^3 + 7x^2 + 16x + 12\)

2. \(f(x) = 2x^4 - 2x^3 - 3x^2 - 7x + 10\)

3. \(f(x) = x^3 - 8x^2 + 11x + 20\)

4. \(f(x) = x^4 + x^3 + 2x^2 + 4x - 8\)

5. \(f(x) = -(x - 4)^2(x + 3)^3(x - 1)(x + 7)\)

6. \(f(x) = -x(x - 2)^2\)

7. \(f(x) = -2x^3 - 6x^2 + 2x + 6\)

8. Identify the number of solutions or zeros. \(y = 2x^5 - 7x^3 - x^2 + x - 4\)

9. What is true about the polynomial function whose graph is shown?

(Circle all that apply)

a) \(f(x) \to -\infty\ \text{as} \ x \to +\infty\)

b) \(f(x) \to +\infty\ \text{as} \ x \to -\infty\)

c) Leading coefficient is positive

d) Degree is even

e) Leading coefficient is negative

f) Degree is odd
10. Which equation is the graph of the polynomial function shown?

   a) \( f(x) = 3x^4 - x^2 + 2 \)
   b) \( f(x) = 3x^3 - x + 7 \)
   c) \( f(x) = -2x^4 + x^2 - 1 \)
   d) \( f(x) = -2x^3 + x^2 + 2 \)

11. Write the equation in factored form whose graph is shown below.

   CHAPTER 6

   Evaluate the expression without using a calculator.

   1. \((16)^\frac{3}{4}\)  
   2. \((\sqrt[3]{-64})^2\)  
   3. \((-1000)^\frac{2}{3}\)  
   4. \((\sqrt{25})^2\)

   5. \((81)^\frac{3}{4}\)  
   6. \((\sqrt[3]{-27})^2\)  
   7. \((16)^{-\frac{3}{4}}\)  
   8. \((\sqrt{-1000})^2\)
Solve: (check for extraneous solutions)

1. \(-2(2x + 3)^3 = 54\)  
2. \(2x^3 + 20 = 36\)  
3. \(24 + (x + 6)^4 = 25\)

4. \(\sqrt{2x + 3} + 2 = 8\)  
5. \(\sqrt[3]{x + 10} = x\)  
6. \(4\sqrt{2x - 8} = 8\)

7. \(\sqrt{7x + 15} = x + 1\)  
8. \(2(x - 2)^{2/3} = 50\)  
9. \(2x^2 = 16\)

10. \(12 - (x + 5)^3 = 76\)

Solve for \(x\)

1. \(4^{3x-1} = \frac{1}{32}\)  
2. \(5^{3x-2} = 5^{-x}\)  
3. \(10^{3x-10} = \left(\frac{1}{100}\right)^{6x-1}\)

4. \(81 \cdot 9^{-2x-2} = 27\)

Find the inverse:

1) \(y = \sqrt[3]{2x - 7}\)  
2) \(y = (x - 3)^2\)

Chapter 7 - Exponential Growth Decay:

\[A = P(1 + r)^t\]  
\[A = Pe^{rt}\]  
\[A = P\left(1 + \frac{r}{n}\right)^{nt}\]

1. Be able to see if an equation is showing exponential growth or decay

2. You deposit $2200 in a bank account. Find the balance after 4 years if the account pays 3% annual interest
   a. compounded quarterly
   b. compounded continuously.

3. A new car that cost $22,000 decreased in value to $6000 in 5 years. Find the average annual rate of depreciation.

4. How much money would you need to deposit now if you wanted to have $60,000.00 in 10 years, if the bank offers 3.5% interest compounded quarterly?
Sequences and Series:

Formulas:  
\[ a_n = a_1 + (n-1)d \]  
\[ a_n = a_1 r^{n-1} \]

\[ S_n = n \left( \frac{a_1 + a_n}{2} \right) \]  
\[ s = \frac{a_1}{1-r} \]  
\[ s_n = a \left( \frac{1-r^n}{1-r} \right) \]

1. Write a rule for the sequence. Find the 5th term of each sequence. The sequences might be arithmetic, geometric, or neither.

a. -2, 2, 6, .......

b. 30, 60, 90, .......

c. 24, 27, 81, .......

d. 1, -3, 9, -27, .......

e. 1, -3, 9, -27, .......

2. Put the following sequences in summation form and then find the sum

a. 8 + 12 + 16 + ... + 116

b. -1, 0.2, -0.04,......

3. Find the sum of the following series.

\[ \sum_{n=1}^{4} n^2 - 1 \]

a. \[ \sum_{n=3}^{10} 2n - 5 \]

b. \[ \sum_{n=1}^{6} 2 \left( \frac{2}{3} \right)^{n-1} \]

c. \[ \sum_{n=1}^{\infty} 2 \left( \frac{1}{3} \right)^{n-1} \]
Algebra 31 Final Exam Review  (Trig, Prob., and Stats)

Trigonometry Review

1. Find the Quadrant in which each angle lies, find the angle’s reference angle, and determine the sine, cosine and tangent for each angle. List two other angles (one positive and one negative) that are co-terminal to the given angle (terminal ray that ends in the same spot).

<table>
<thead>
<tr>
<th>Degrees</th>
<th>Radians</th>
<th>Cos θ</th>
<th>Sin θ</th>
<th>Tan θ</th>
<th>(+)Coterminal</th>
<th>(-)Coterminal</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>$\frac{\pi}{2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>540°</td>
<td>$\frac{5\pi}{6}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-60°</td>
<td>$\frac{\pi}{6}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. If \( \sin \theta = \frac{\sqrt{2}}{5} \) and \( 90 < \theta < 180 \) what is \( \tan \theta \) and \( \cos \theta \)?

3. If \( \cos \theta = -\frac{5}{13} \) and \( 180 < \theta < 270 \) what is \( \sin \theta \) and \( \tan \theta \)?

4. If \( \sin \theta = \frac{8}{17} \) what are the possible values of \( \cos \theta \) and \( \tan \theta \)?

5. If \( \sin \theta = \frac{\sqrt{5}}{5} \) and \( 90 < \theta < 180 \) what are the possible values of \( \cos \theta \) and \( \tan \theta \)?

6. Give a co-terminal angle for \( -\frac{\pi}{3} \) over the domain \( 0 < \theta < 2\pi \).
7. Graph \( f(x) = -2\sin(x) - 1 \). Label y axis with appropriate scale, list points for local minimums and maximums, write the equation for the midline, list the period, and domain and range.

Domain: _____________________________ Range: _________________________ Amplitude: ________________

Phase Shift: _________________________ Vertical Slide: _________________ Period: ________________

Maximum: ___________________________ Minimum: _____________________

8. Graph \( f(x) = 3\cos\left(x - \frac{\pi}{4}\right) \). Label both axes with appropriate scales, list points for local minimums and maximums, write the equation for the midline, list the period, and domain and range.

Domain: _____________________________ Range: _________________________ Amplitude: ________________

Phase Shift: _________________________ Vertical Slide: _________________ Period: ________________

Maximum: ___________________________ Minimum: _____________________
9. Graph \( f(x) = 2\sin(x) - 1 \). Label y axis with appropriate scale, list points for local minimums and maximums, write the equation for the midline, list the period, and domain and range.

\[
\begin{align*}
\text{Domain: } & \quad \text{Range: } \quad \text{Amplitude: } \\
\text{Phase Shift: } & \quad \text{Vertical Slide: } \quad \text{Period: } \\
\text{Maximum: } & \quad \text{Minimum: }
\end{align*}
\]

10. Write a sine or cosine equation for the periodic function.

a. Equation: ____________________________  
b. Equation: ____________________________  
c. Equation: ____________________________
Probability Review

1. A survey of 90 Fairfield Ludlowe High School students was taken and the survey asked them about “what is their favorite place to eat?” Thirty-nine students like Señor Salsa (S), 24 like Centro (C), and 35 like Panera Bread (P). 19 like Señor Salsa and Panera, 15 liked Señor Salsa and Centro, and 5 liked Centro and Panera. Three students liked all three restaurants. Create a Venn diagram in order to answer the questions below:

a) What is the Probability that they like Senor Salsa or Panera?
b) What is the Probability that they like Senor Salsa and Centro?
c) What is the probability that they don’t like Centro?
d) What is the $P(S^c \cap P^c \cap C^c)$?
e) What is the $P(S \cap C \cap P^c)$?
f) What is the $P(S \cup C \cup P)$?
g) What is the probability that a randomly selected student only likes Señor Salsa?

2. A company is performing a market test on two different cereals, A and B, for its new product line. Out of 150 people who try the cereals, 60 like cereal A, 95 like cereal B, and 25 like both cereals.

a) What is the probability that a person likes cereal A or cereal B?
b) What is the probability that a person does not like either cereal?
c) What does $P(A^c)$ mean in words in context of the problem and what is the value of $P(A^c)$?

3. A jar contains 15 red candies, 12 blue candies, and 16 yellow candies.

a) What is the probability of selecting a yellow candy?
b) What is the probability of selecting 2 blue candies in a row if you must put the candy back before taking the next piece?
c) What is the probability of selecting 3 red candies in a row if you can eat the 1st and 2nd candies before taking the 3rd piece?

4. Is rolling a dice and getting a 4, then rolling again and getting a 6 an example of independent or dependent events? Explain.
5. A group of students were surveyed on their preference for pirate or ninja movies. The data is summarized below. P = prefers pirates, N = prefers ninjas, F = female, M = male.

<table>
<thead>
<tr>
<th></th>
<th>Pirates</th>
<th>Ninjas</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>18</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>22</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Fill in the missing values in the table.
b) How many students were surveyed?
c) Find the probability that their preference is a pirate movie, given that the student is a male.
d) What is the probability that the student is a female and prefers ninja movies?
e) What is the probability that a random student prefers ninja movies?

6. Fill in the two-way table with the following information about the Junior class at a high school. There are a total of 430 students in the Junior class, 40% have attended a sporting event at the school and 50% have attended a school dance. 98 students in the Junior class have attended both a sporting event and a school dance. Fill in the missing values in the table.

<table>
<thead>
<tr>
<th>Attended a Sporting Event</th>
<th>Attended a Dance</th>
<th>Has not attended a Dance</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has not attended a Sporting Event</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. You flip a coin one time and a 6 sided dice.
   a) What is the sample space?
   b) P(head and the number 4)
   c) P(head or the number 4)
   d) P(rolling an even number)
   e) P(head or rolling an odd number)
   f) P(head and rolling an even number)
7) Oostburg College has a rather large marching band. Engineering majors were heard bragging that students majoring in engineering are more likely to be involved in the marching band than students from other majors. If the above claim is accurate, does that mean that most of the band is engineering students? Explain your answer.

The following graph was prepared to investigate the above claim.

![Bar graph showing frequency of students in the marching band by major]

- **a)** Based on the graph, complete the following two-way frequency table:

<table>
<thead>
<tr>
<th></th>
<th>Marching Band</th>
<th>Not in Marching Band</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engineering Major</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not an Engineering Major</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **b)** Based on the completed two-way frequency table, determine the following and explain how you got your answer. Show fraction and decimal (round to three decimal places)

  i. The probability that a randomly selected student is in the marching band.
  ii. The probability that a randomly selected student is an engineering major.
  iii. The probability that a randomly selected student is in the marching band and an engineering major.
  iv. The probability that a randomly selected student is in the marching band and not an engineering major.
  v. A randomly selected student is majoring in engineering. What is the probability that this student is in the marching band?
  vi. A randomly selected student is not majoring in engineering. What is the probability that this student is in the marching band?
  vii. The claim that started this investigation was that students majoring in engineering are more likely to be in the marching band than students from other majors. Describe the conditional probabilities that would be used to determine if this claim is accurate.
1. The performance reviews for several employees at a company are shown in the box-and-whisker plot below. What is the range of the performance review scores?

<table>
<thead>
<tr>
<th>Performance Reviews</th>
</tr>
</thead>
<tbody>
<tr>
<td>52 56 60 64 68 72 76 80 84 88 92 96 100</td>
</tr>
</tbody>
</table>

- a. 31
- b. 36
- c. 20
- d. 34

2. Which box-and-whisker plot represents a situation where 25% of the data is 200 or less?

- a. 0 100 200 300 400
- b. 0 100 200 300 400
- c. 0 100 200 300 400
- d. 0 100 200 300 400

3. The two box-and-whisker plots below show the average monthly temperatures for Boston and Seattle. What do the interquartile ranges tell you about the two cities?

- a) Boston has more consistent temperatures.
- b) Seattle has more consistent temperatures.
- c) Overall Seattle is warmer than Boston.
- d) Overall Boston has a colder than Seattle.
4. In a recent survey of 1500 randomly selected U.S. adults, 68% of the respondents agreed with the statement, “I should exercise more than I do.” For this study, state one source of potential bias and how it would affect the estimate of the proportion of adults who would agree with the statement, “I should exercise more than I do.” The number 68% is

   a) a margin of error   b) a parameter   c) a reliability   d) a statistic

5. A researcher gathers data on how long teenagers spend on individual cell phone calls (in number of minutes). Suppose the research determines that these calls have a mean 45 min. and standard deviation 7 min. The researcher also claims that the distribution of the call lengths follows a normal distribution.

   Label your horizontal axis with the mean and the values 1, 2 and 3 standard deviations above and below the mean. Shade the area that represents one standard deviations within the mean.

6. Use the data for average daily water usage of a family during the past 12 months. Find the mean and the standard deviation of the data. Within how many standard deviations from the mean do all the values fall?

   126 118 134 545 150 480 442 310 124 149 233 367

7. Susan keeps track of the number of tickets sold for each play presented at the Community Theater. Within how many standard deviations from the mean do all the values fall?

   135, 71, 69, 80, 158, 152, 161, 96, 122, 118, 87, 85
8. A town planning board wants to estimate the average value of houses in Fairfield. The board randomly selects 500 houses from different areas of the town and uses town records to calculate the median house value to be $502,200.

   a) Is this an experiment, survey or observational study? **Explain** your choice.

   b) What is the sample and what is the population?

   c) Is number $502,200 a statistic or a parameter? **Explain**.

9. An education official wants to estimate the proportion of adults aged 18 or older who had read at least one book during the previous year. A random sample of 1006 adults aged 18 or older is obtained, and 835 of those adults had read at least one book during the previous year.

   Population:       Sample:

   Parameter:       Statistic:

For questions #10 – 13: **Explain** whether each situation is an experiment, observational study, or survey.

10. The research department of a retail company plans to conduct a study to determine whether a dye used on a new T-shirt will begin fading before 50 washes.

11. A literacy group wants to determine whether high school students that participated in a recent national reading program had higher standardized test scores than high school students that did not participate in the program.

12. A group of high school students is randomly selected and asked to complete the form shown.

13. An advertising company wants to test a new logo design. They randomly select 20 participants and watch them discuss the logo.
14. A recent survey by alumni of a major university indicated that the average salary of 11,500 of its 225,000 graduates was $100,000. Does this value describe a population parameter or a sample statistic? Explain.

15. The average salary of all General Motors workers is $30,500. Does this value describe a population parameter or a sample statistic? Explain.

16. In March of this year, the postmaster for Stamford, CT wanted to plan for the expected crowds filing their tax returns at the last minute. To do this, she decided to estimate the percentage of tax filers who planned to file their returns on April 15. Using a random-digit dialing technique, a group of 4000 tax filers were interviewed, revealing that 350 of them said it was likely that they would not file until the last day.

**In this problem, identify the:**

a) population of interest: __________________________

b) parameter: __________________________

c) sample: __________________________

d) sample statistic: __________________________

e) margin of error: __________

f) What is the interval for the percentage of tax filers who would file on the last day? __________