

## 2016-2017 Pre-Calculus 40 Final Outline/Review:

Non-Calculator Section: 16 multiple choice (32 pts) and 6 open ended (24 pts).

Calculator Section: 8 multiple choice (16 pts) and 11 open ended (36 pts).

### First Semester Topics:

- Logarithmics and Exponentials (6): Rewrite, evaluate, solve and investment
- Polynomials (2): Solve using Fundamental Theorem of Algebra and x-intercepts
- Rationals (2): Simplify, Multiply, Add and vertical asymptotes.

### Second Semester Topics:

#### Trigonometry:

- Right Triangle Trig (1)
- Area of triangles (2): formulas given
- Evaluate Unit Circle Values (4): including sin, cos, tan, csc, sec and cot - also: given one trig - evaluate others, using sum and difference (formulas given).
- Trigonometric Inverses (2): single and composition
- Solve Triangles (2): using law of Sines and Law of Cosines (given)
- Solving Equations (2): involving sine, cosine, tangent, cosecant, secant and cotangent
- Establishing Trig Identities (1): some identities are given.

#### Parametric, Polar and Vectors:

- Convert between Parametric and Rectangular (1)
- Using parametric to create equations of a projectile, formulas and gravity given (1): Calculate max height, time at max height, and distance and time when landed.
- Convert between Polar and Rectangular (4): Points and equations
- Graph Polar(1): calculator section
- Vectors (5): determine vector both given points and given magnitude and direction, determine Magnitude and properties of Vectors (sum, difference, scalar multiplication).

#### WORD PROBLEM

- Force (1): Find magnitude of horizontal and vertical components.

#### Other

- Matrices (3) - scalar multiplication with addition/subtraction (noncalc) and multiply (calc), solve system (calc)
- Simplify (Rationalize) a complex function in rational form (1).

## First Semester Topics:

### NONCALCULATOR:

1. Determine if the following equations are polynomials. If they are polynomials, determine the degree, leading coefficient and constant term.

a.  $f(x) = 3x$

b.  $g(x) = 4x^5 - 3x^2 + 2$

c.  $h(x) = \sqrt{5x^3 - 2x^2 + 12}$

2. Determine whether the  $x$  value is a solution to the equation  $0 = x^4 + 6x^3 + 9x^2 + 24x + 20$

a.  $x = 2$

b.  $x = -1$

c.  $x = 0$

3. Determine all real solutions of  $2x^3 - x^2 - 7x + 6 = 0$ . NONCALCULATOR! Show Factored Form.

4. Determine all real solutions of  $x^3 + 3x^2 - 4x - 12 = 0$ . NONCALCULATOR! Show Factored Form.

5. Simplify the Rational Expressions:

a.  $\frac{3p^2 - 9p - 30}{6p^2 + 6p - 12}$

b.  $\frac{x^2 + 8x + 15}{2x^2 - 18}$

c.  $\frac{3x^2 - 4x - 4}{x^2 - 4}$

6. Determine the vertical asymptote(s) of the function

a.  $f(x) = \frac{x^2-4}{x^2-2x-8}$

b.  $h(x) = \frac{x+4}{x^2+x-12}$

c.  $\frac{5x+1}{x^2-3x}$

7. Evaluate (non-calculator):

a.  $\log_3 81$

b.  $\log_2 \frac{1}{16}$

c.  $\ln e^{-4}$

8. Solve the equation (non-calculator):

a.  $32^{x+1} = 8^{5x}$

b.  $16^{2x+1} = 64^{3x+2}$

c.  $\log_2 4 + \log_2(x-1) = \log_2(x+1)$

d.  $\ln(3x+1) + 2\ln(3) = \ln(x^2+19x)$

e.  $\log_2 16 = x+3$

f.  $\log_3 27 = 2x+1$

g.  $\log_4(3x+4) = 3$

h.  $5^x = 125$

i.  $3^{x+1} = 81$

9. Simplify the expression into a+bi form:

a.  $(5+i)(3-2i)$

b.  $\frac{6+2i}{3-2i}$

c.  $\frac{1-5i}{4+i}$

**CALCULATOR:**

10. Paula invests \$250 at 3.75% compounded monthly. Determine the amount in her account after 15 years.
11. Frederico invests \$100 at 3.75% compounded continuously. How long will it take for him to double his investment?
12. What principal invested at 4.65% continuously for 10 years will yield \$12,000? Round to 2 decimal places.
13. The decay of a 50 mg sample of radium is given by the equation  $R(t) = 50e^{-0.000433t}$ , where  $t$  is time in years and  $R(t)$  is the amount of radium at a given point in time.
- How long will it take for the sample to decay to 10 mg?
  - What is the half life of radium?
14. The number of articles making up an on-line open-content encyclopedia increased exponentially during the first few years. The number of articles,  $A(t)$ ,  $t$  years after 2001, can be modeled by  $A(t) = 16,198(2.13)^t$ .
- According to this model, how many articles made up the encyclopedia in 2001?
  - At what rate is the number of articles increasing?
  - During which year did the encyclopedia reach one million articles?
  - Predict the number of articles there will be at the beginning of 2018.

## Second Semester Topics:

### NONCALCULATOR:

15. Write each degree measure in radians as a multiple of  $\pi$  and each radian measure in degrees.

a.  $136^\circ$

b.  $-45^\circ$

c.  $\frac{3\pi}{4}$

d.  $-\frac{5\pi}{6}$

16. Find the exact value of each trigonometric function, if defined. If not defined, write undefined.

a.  $\tan(-45^\circ)$

b.  $\cos\left(\frac{3\pi}{2}\right)$

c.  $\csc\left(\frac{5\pi}{6}\right)$

d.  $\sin(900^\circ)$

e.  $\sec\left(\frac{\pi}{2}\right)$

17. Determine all solutions to the equation on  $[0, 2\pi)$

a.  $4\tan(x) - 7 = 3\tan(x) - 6$

b.  $9 + \sin^2(x) = 10$

c.  $7\cos(x) = 5\cos(x) + \sqrt{3}$

d.  $5\sin(x) + 2 = \sin(x)$

18. Find the exact value of each of the following WITHOUT using a calculator!

a.  $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =$

b.  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

c.  $\arccos\left(-\frac{\sqrt{2}}{2}\right) =$

d.  $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right) =$

e.  $\sin^{-1}(\sin(-\pi)) =$

f.  $\cos\left(\arctan\left(\frac{5}{2}\right)\right) =$

g.  $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right) =$

h.  $\cos\left(\sin^{-1}\left(\frac{3}{8}\right)\right) =$

19. Find the exact value of each trigonometric function.

a.  $\cos 15^\circ$

b.  $\sin \frac{19\pi}{12}$

c.  $\tan 255^\circ$

d.  $\cos 25^\circ \cos 35^\circ - \sin 25^\circ \sin 35^\circ$

e.  $\sin 135^\circ + \tan 60^\circ$

f.  $\cos(-120^\circ) - \sin(315^\circ)$

20. Establish the below identities

a.  $\sec^2 x (1 - \cos^2 x) = \tan^2 x$

b.  $(\sec^2 x - 1) \cos^2 x = \sin^2 x$

c.  $\sin x (\csc x - \sin x) = \cos^2 x$

21. Let  $\overrightarrow{AB}$  be the vector with initial point  $A(10, -4)$  and terminal point  $B(-1, -3)$ . Write  $\overrightarrow{AB}$  as a linear combination of the vectors  $\mathbf{i}$  and  $\mathbf{j}$ .

22. Find the component form of  $\overrightarrow{AB}$  with initial point  $A(-12, 7)$  and terminal point  $B(8, -2)$ .

23. Given  $\mathbf{r} = \langle 3, 9 \rangle$  and  $\mathbf{s} = \langle -3, 6 \rangle$

a.  $2\mathbf{r} - \mathbf{s}$

b.  $5\mathbf{r} - 2\mathbf{s}$

c.  $\mathbf{r} + 2\mathbf{s}$

24. Write the following parametric equations in rectangular form:

a.  $x = t - 1, y = 2t^2 + 6$

b.  $x = 4 \cos \theta, y = 2 \sin \theta$

c.  $x = 3t + 9, y = t^2 - 7$

d.  $x = t^2 + 1, y = 4t + 3$

25. Find the rectangular coordinates of the polar points:

a.  $(4, 120^\circ)$

b.  $(-2, 3\pi/4)$

c.  $(3, -\pi/3)$

26. Find one set of polar coordinates for the following rectangular coordinates if  $r > 0$ :

a.  $(6, 6)$

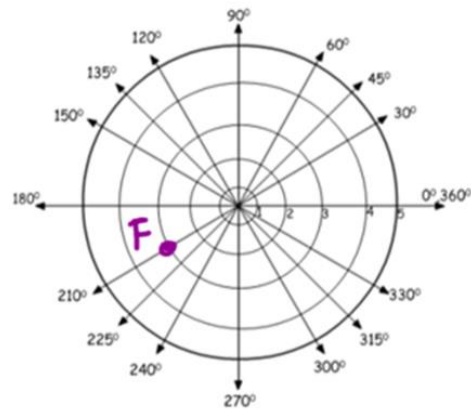
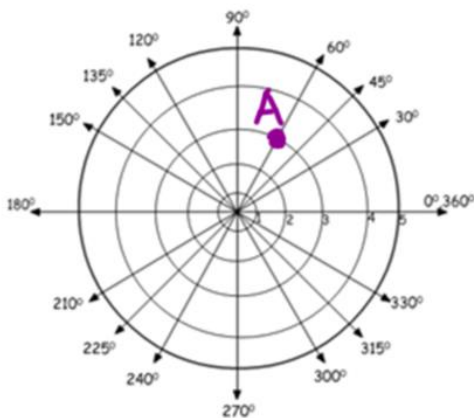
b.  $(-2\sqrt{3}, 2)$

c.  $(-5, -5\sqrt{3})$

27. Name the polar coordinates of points A and F graphed below if:

a.  $r > 0$  and  $0 \leq \theta \leq 360^\circ$

b.  $r < 0$  and  $0 \leq \theta \leq 360^\circ$



28. Write the polar equations in rectangular form:

a.  $r = 5$

b.  $r = 2\csc\theta$

c.  $\theta = \frac{5\pi}{6}$

29. Write the rectangular equations in polar form:

a.  $x^2 + y^2 = 16$

b.  $y = 4$

c.  $y = -x$

d.  $x = 6$

30. Find the component form of  $\overrightarrow{AB}$  given

a.  $|v| = 12$  and direction angle  $\theta = \frac{5\pi}{3}$

b.  $|v| = 5$  and direction angle  $\theta = 120^\circ$

31. Given the matrices below, what is  $2A + 3B$ ? (No Calculator)

$$A = \begin{bmatrix} 5 & 0 & 9 \\ -1 & 4 & -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 & -4 \\ 0 & 3 & -9 \end{bmatrix}$$



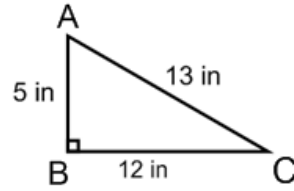
**CALCULATOR:**

32. In  $\triangle ABC$  below, find the following values.

a. Angle  $A =$  \_\_\_\_\_

b. Angle  $C =$  \_\_\_\_\_

c.  $\tan A =$  \_\_\_\_\_



33. A pilot needs to begin his descent when his plane is 7.5 km above ground and 200 km straight to the airport. At what angle should his descent be so that he can fly in a straight line from the point of initial descent to the ground? How much ground will he pass from the point of initial descent until he touches down at the airport?

34. If a building is 423 ft tall and the angle from its shadow to the top of the building is  $43^\circ$ , determine the length of the shadow.

35. A blimp was flying above Fairfield the other day at an altitude of 425 meters. Emily was in the blimp and she saw the high school. She calculated the angle of depression from the blimp to the entrance of the high school was about  $48^\circ$ . If she dropped a rock out of the blimp and the rock fell straight to the ground, how far away from the high school would the rock land? Round your answer to the nearest meter.

36. Given a triangle with the following dimensions, solve for the remaining sides and angles.

a.  $a = 11\text{ cm}$ ,  $b = 6\text{ cm}$ ,  $A = 22^\circ$

b.  $a = 13\text{ m}$ ,  $b = 12\text{ m}$ ,  $c = 8\text{ m}$

c.  $a = 9\text{ cm}$ ,  $b = 10\text{ cm}$ ,  $C = 42^\circ$

d.  $a = 5\text{ cm}$ ,  $A = 36^\circ$ ,  $B = 42^\circ$

e.  $A = 63^\circ$ ,  $a = 18\text{ in}$ ,  $b = 25\text{ in}$

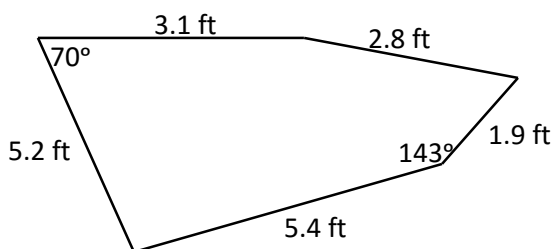
f.  $A = 20^\circ$ ,  $a = 4\text{ mm}$ ,  $b = 6\text{ mm}$

37. Determine the area of each triangle to the nearest tenth.

a.  $A = 95^\circ$ ,  $b = 12\text{ m}$ ,  $c = 18\text{ m}$

b.  $a = 44$ ,  $b = 47$ ,  $c = 53$

38. Mrs. Shannon wants a uniquely shaped blanket for the hours she will be lounging at the beach this summer. If the blanket is in the shape below determine the size of the blanket.



39. A plane takes off at 220 miles per hour at an angle of  $51^\circ$  with the ground. Find the magnitude of the horizontal and vertical components of its velocity. Round to the nearest tenth.

40. Charles leaves his apartment and walks  $55^\circ$  east of north for 1000 feet and then walks 300 feet due north to go bowling. Write a vector to represent each stage of Charles' trip. How far and at what quadrant bearing is Charles from his apartment when he arrives at the bowling alley?

41. Determine the direction angle and magnitude of the following vectors.

a.  $\langle 2, -2 \rangle$

b.  $\langle 0, 7 \rangle$

c.  $\langle -2, 5 \rangle$

42. Find the component form of  $\overrightarrow{AB}$  given  $|v| = 12$  and direction angle  $\theta = 120^\circ$ .

43. Charles is pulling a wagon with a force of 315 Newtons at angle of  $37^\circ$  with the ground. Draw a diagram showing the vertical and horizontal components and then find the magnitudes of horizontal and vertical components of the force. Round to the nearest tenth.

Horizontal Component: \_\_\_\_\_

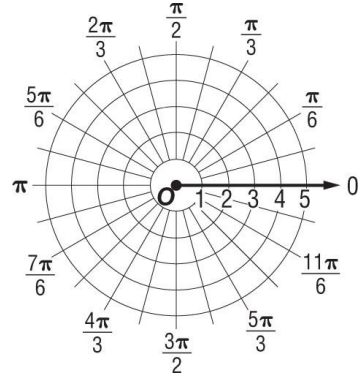
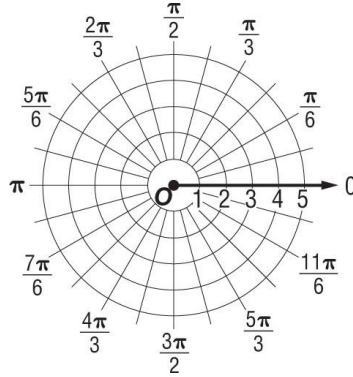
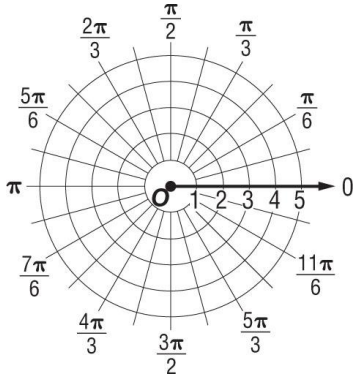
Vertical Component: \_\_\_\_\_

44. Name the Graph of the polar equation and Sketch its graph using a Graphing Calculator.

a.  $r = 2 - 2\sin\theta$

b.  $r = 3 + 2\cos\theta$

c.  $r = 4\cos(3\theta)$



45. Suppose Mr. Ebling kicks a soccer ball with an initial velocity of 150 feet per second at an angle of  $30^\circ$  to the horizontal. Round all answers to the nearest tenth.

a. Write a set of parametric equations that describe the position of the ball as a function of time.

b. How high is the ball after 1 second?

c. How long is the golf ball in the air?

d. When is the ball at its maximum height?

e. What is the maximum height of the golf ball?

f. How far away did the golf ball land?

46. Determine  $AB$  for the matrices below. Use of a calculator is permitted.

$$A = \begin{bmatrix} 6 & 9 & 1 \\ 4 & 0 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -2 \\ 4 & 3 \\ 5 & -5 \end{bmatrix}$$

47. Write a matrix for the system of equations. Then solve for each variable using your calculator. Round to two decimal places.

$$2x + 7y - 2z = 5$$

$$3y - x = 6z + 12$$

$$-3z = 2x + 14 - 4y$$

Answer Key:

1a. Yes, 1<sup>st</sup> degree, leading coefficient is 3, no constant

1b. Yes, 5<sup>th</sup> degree, leading coefficient is 4, constant is 2

1c. Not a polynomial

2a. No

2b. yes

2c. No

3.  $x = -2, 1, \text{ and } 3/2$

4.  $x = -3, -2, \text{ and } 2$

5a.  $\frac{p-5}{2p-2}$

5b.  $\frac{x+5}{2x-6}$

5c.  $\frac{3x+2}{x+2}$

6a.  $x = 4$

6b.  $x = 3$

6c.  $x = 0 \text{ and } x = 3$

7a.  $x = 4$

7b.  $x = -4$

7c.  $x = -4$

8a.  $x = 1/2$

8b.  $x = -4/5$

8c.  $x = \frac{5}{3}$

8d.  $x = 9$  (-1 is an extraneous solution)

8e.  $x = 1$

8f.  $x = 1$

8g.  $x = 20$

8h.  $x = 3$

8i.  $x = 3$

9a.  $17 - 7i$

9b.  $\frac{14}{13} + \frac{18}{13}i$

9c.  $-\frac{1}{17} - \frac{21}{17}i$

10. \$434.27

11. 18.48 years

12. \$7,537.62

13a. 3,717 years

13b. 1600.8 years

14a. 16,198

14b. 113% or 75.61%

14c. 2006

14d. 6,193,190,000

15a.  $\frac{34\pi}{45}$

15b.  $-\frac{\pi}{4}$

15c.  $135^\circ$

15d.  $-150^\circ$

16a. -1

16b. 0

16c. 2

16d. 0

16e. undefined

17a.  $x = \frac{\pi}{4}, \frac{5\pi}{4}$

17b.  $x = \frac{\pi}{2}, \frac{3\pi}{2}$

17c.  $x = \frac{\pi}{6}, \frac{11\pi}{6}$

17d.  $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

18a.  $-30^\circ \text{ or } -\frac{\pi}{6}$

18b.  $60^\circ \text{ or } \frac{\pi}{3}$

18c.  $135^\circ \text{ or } \frac{3\pi}{4}$

18d.  $-60^\circ \text{ or } -\frac{\pi}{3}$

18e. 0

18f.  $\frac{2\sqrt{29}}{29}$

18g.  $\frac{4}{5}$

18h.  $\frac{\sqrt{55}}{8}$

19a.  $\frac{\sqrt{6} + \sqrt{2}}{4}$

19b.  $\frac{-\sqrt{6} - \sqrt{2}}{4}$

19c.  $\frac{\sqrt{3} + 3}{3 - \sqrt{3}}$  or  $\frac{-\sqrt{3} - 1}{1 - \sqrt{3}}$

19d.  $\frac{1}{2}$

19e.  $\frac{\sqrt{2} + 2\sqrt{3}}{2}$

19f.  $\frac{\sqrt{2} - 1}{2}$

20a.  $\tan^2 x = \tan^2 x$

20b.  $\sin^2 x = \sin^2 x$

20c.  $\cos^2 x = \cos^2 x$

21.  $-11i + j$

22.  $\langle 20, -9 \rangle$

23a.  $\langle 9, 12 \rangle$

23b.  $\langle 21, 33 \rangle$

23c.  $\langle -3, 21 \rangle$

24a.  $y = 2x^2 + 4x + 8$

24b.  $\frac{x^2}{16} + \frac{y^2}{4} = 1$

24c.  $y = \frac{1}{9}x^2 - 2x + 2$

24d.  $y = 3 \pm 4\sqrt{x-1}$

25a.  $(-2, 2\sqrt{3})$

25b.  $(\sqrt{2}, -\sqrt{2})$

25c.  $(\frac{3}{2}, -\frac{3\sqrt{3}}{2})$  or  $(1.5, -1.5\sqrt{3})$

26a.  $(6\sqrt{2}, \frac{\pi}{4})$

26b.  $(4, \frac{5\pi}{6})$  or  $(-4, \frac{11\pi}{6})$

26c.  $(10, \frac{4\pi}{3})$

27. Part A a.  $(3, 60^\circ)$  b.  $(-3, 240^\circ)$

27. Part B a.  $(3, 210^\circ)$  b.  $(-3, 30^\circ)$

28a.  $x^2 + y^2 = 25$

28b.  $y = 2$

28c.  $y = -\frac{\sqrt{3}}{3}x$

29a.  $r = 4$

29b.  $r = 4\csc\theta$

29c.  $\theta = \frac{3\pi}{4}$

29d.  $r = 6\sec\theta$

30a.  $\langle 6, -6\sqrt{3} \rangle$

30b.  $\langle -\frac{5}{2}, \frac{5\sqrt{3}}{2} \rangle$

31.  $\begin{bmatrix} 16 & 3 & 6 \\ -2 & 17 & -39 \end{bmatrix}$

32a.  $67.38^\circ$ ,

32b.  $22.62^\circ$

32c.  $\frac{12}{5}$

33. Angle of depression:  $2.15^\circ$ , ground covered: 199.86 km

34. Length of shadow: 453.62 feet

35.  $382.67 = 383$  meters

36a.  $B = 11.79^\circ$   
 $C = 146.21^\circ$   
 $c = 16.33 \text{ cm}$

36b.  $A = 78.28^\circ$   
 $B = 64.67^\circ$   
 $C = 37.05^\circ$

36c.  $A = 61.1^\circ$   
 $B = 76.9^\circ$   
 $c = 6.87 \text{ cm}$

36d.  $C = 102^\circ$   
 $b = 5.69 \text{ km}$   
 $c = 8.32 \text{ cm}$

36e. Not Possible

36f.  $B = 30.87^\circ$   
 $C = 129.13^\circ$   
 $C = 9.07 \text{ cm}$

AND  $B = 149.13^\circ$   
 $C = 10.87^\circ$   
 $C = 2.21 \text{ cm}$

37a. 107.59 meters squared

37b. 978.57 units squared

38. 16.56 feet squared

39. horizontal component: 138.5, vertical component: 171.0

40. 1000 feet at 55 degrees east of north:  $\langle 819.15, 573.58 \rangle$ , 300 feet due north:  $\langle 0, 300 \rangle$ , resultant:  $\langle 819.15, 873.58 \rangle$

Distance and bearing from home when at bowling alley: 1197.56 feet at 46.84 degrees above the horizontal or  $N43.16^\circ E$ .

41a. Magnitude:  $2\sqrt{2}$ , Direction Angle:  $315^\circ$     41b. Magnitude: 7, Direction Angle:  $\frac{\pi}{2}$

41c. Magnitude:  $\sqrt{29}$ , Direction Angle:  $111.8^\circ$

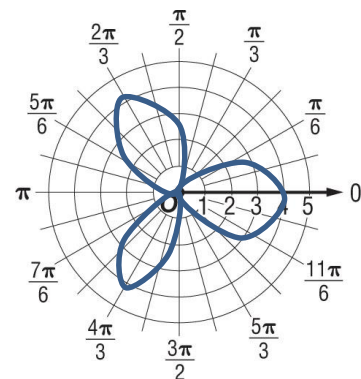
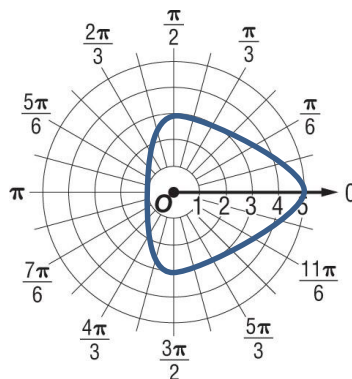
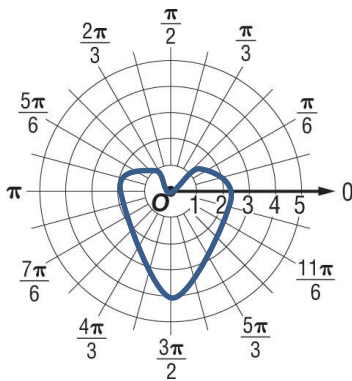
42.  $\langle -6, 6\sqrt{3} \rangle$

43.  $\langle 251.6, 189.6 \rangle$

44a.  $r = 2 - 2\sin\theta$   
 Cardoid

44b.  $r = 3 + 2\cos\theta$   
 Limacon

44c.  $r = 4\cos(3\theta)$   
 Rose



45a.  $x = (150\cos 30^\circ)t$  or  $(75\sqrt{3})t$ ,  $y = (150\sin 30^\circ)t - \frac{1}{2}(32)t^2$  or  $-16t^2 + 75t$

45b. 59 feet

45f. 608.92 feet

45c. 4.6875 seconds

45d. 2.34 seconds

46.  $\begin{bmatrix} 53 & 10 \\ -7 & 7 \end{bmatrix}$

45e. 87.89 feet

47a.  $\begin{bmatrix} 2 & 7 & -2 & : & 5 \\ -1 & 3 & -6 & : & 12 \\ -2 & 4 & -3 & : & 14 \end{bmatrix}$

b. solution:  $x = -3.07$ ,  $y = 1.36$ ,  $z = -.81$