Algebra 32 Midterm Review Packet

Formula you will receive on the Midterm:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Name: _____

Teacher: _____

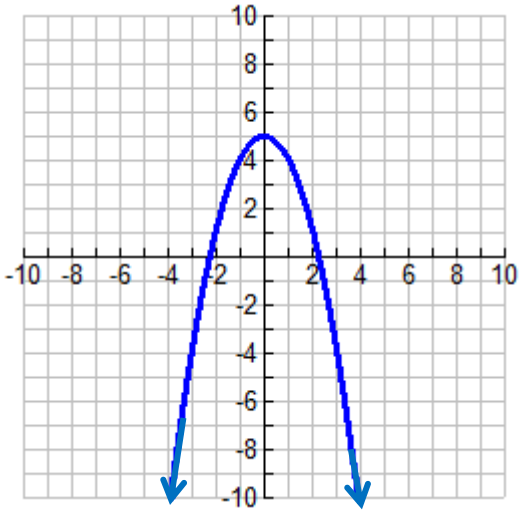
Day/Period: _____

Date of Midterm: _____

Functions:

- Vocabulary:
  - Domain (Input) & Range (Output)
  - Increasing & Decreasing
  - Minimum & Maximum
- Types of Functions:
  - Linear, Quadratic (parabola), Cubic, Quartic, Polynomial
  - Piecewise
  - Absolute Value
- Evaluate functions for values without a calculator by direct or synthetic substitution
- Transformations on Functions
  - Translations (left, right, up, down)
  - Reflections (over x-axis or y-axis)
  - Dilations (vertical stretch or shrink)

1) Answer the following questions about the graph *f(x).*



   a) Use interval notation:

      Domain: _____

      Range: _____

      On what interval is f(x) decreasing? _____

      On what interval is f(x) increasing? _____

   b) What is the maximum? _____

   c) Estimate the x-intercepts: _____ & _____

   d) Find the values on *f(x)*:
      a. f(1)
      b. f(0)
      c. f(x) = -4

2) Use the function $g(x) = x^4 - 3x^2$ to answer the following questions without a calculator:
   a) Find the value of *g(x)* when x = 1
   b) Find the value of *g(x)* when x = -1

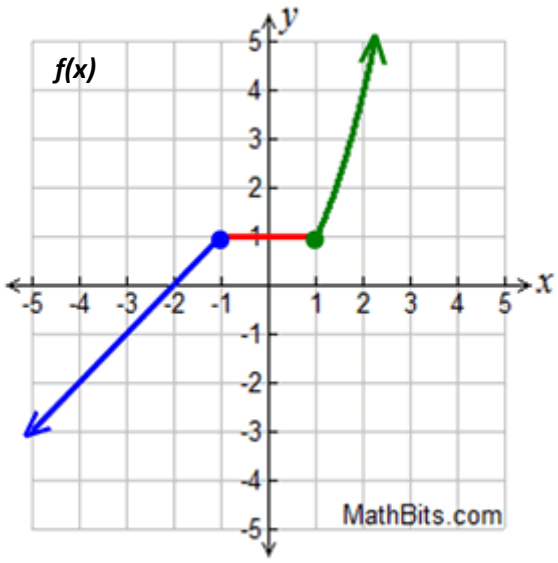
3) Using the function f(x) = 2x – 4, determine the following:
   a) f(2)
   b) f(-3)
   c) f(x - 2)
   d) If f(x) = 8, determine x.
   e) If f(2x – 3) = 10, determine x.

4) Find the value of f(x) = -2x + 8 when
   a) x = 8
   b) f(x) = 12
   c) x = -2
   d) f(x) = 16
   e) x = -½

5) Use the function $h(x) = x^3 + 2x^2 + 4x + 2$ to answer the following questions without a calculator:
   a. Find the value of *h(x)* when x = –1

   b. Find the value of *h(x)* when x = 2

6) Answer the following questions about the piecewise function shown below.



a) Use interval notation:

   Domain: _____

   Range: _____

b) What is the y-intercept? _____

c) What is the x-intercept? _____

d) Estimate the following using the graph:
   a. f(2)
   b. f(-1)
   c. f(x) = -2
   d. f(x) = 4

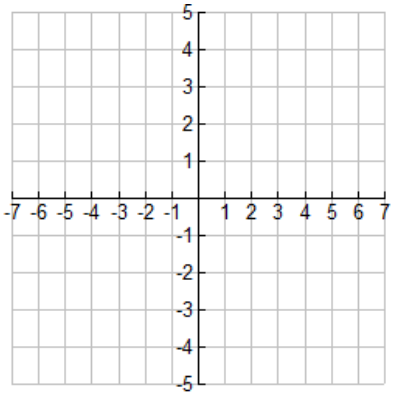
7) Describe the transformations in detail: reflect over y axis, translate left 2, dilation- vertical stretch, etc…
   a) $f(x) + 10$
   b) $f(-x)$
   c) $-f(x)$
   d) $3f(x)$
   e) $f(x + 4)$

8) Describe how the parent functions $g(x) = |x|$, $h(x) = x^2$ and $k(x) = \sqrt{x}$ are transformed.

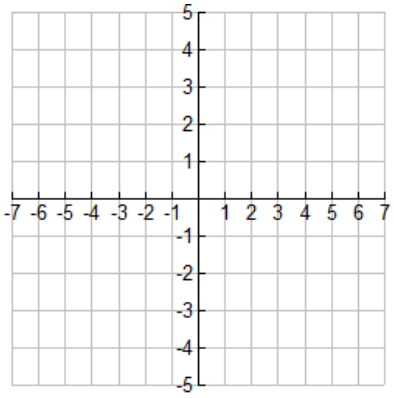
a) $g(x) = |x| - 4$
b) $g(x) = |x| + 6$
c) $g(x) = 3|x|$
d) $g(x) = -2|x|$
e) $g(x) = \frac{1}{2}|x|$
f) $h(x) = -(x)^2$
g) $h(x) = (x)^2 - 4$
h) $h(x) = 4(x)^2$
i) $k(x) = \sqrt{(x+1)}$
j) $k(x) = \sqrt{x-2}$
k) $k(x) = \frac{1}{3}\sqrt{x}$

9) Graph the equations:

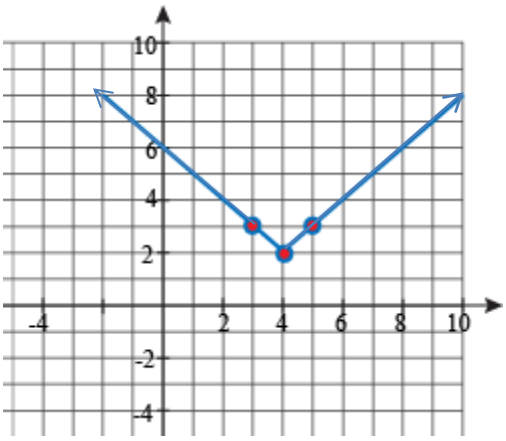
a) $y = |x - 1|$



b) $y = |x| + 3$



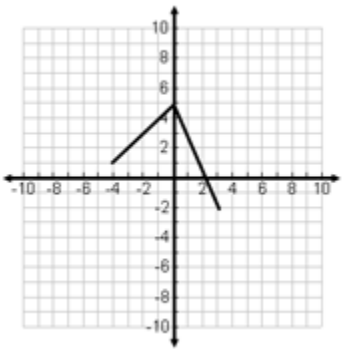
10) Which of the options is the equation for the graph shown below?



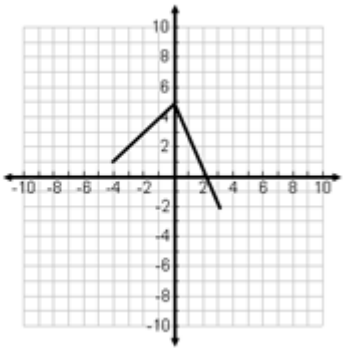
a) $y = |x - 4| - 2$
b) $y = |x - 4| + 2$
c) $y = |x + 4| + 2$
d) $y = |x + 4| - 2$

11) Perform the given transformation:

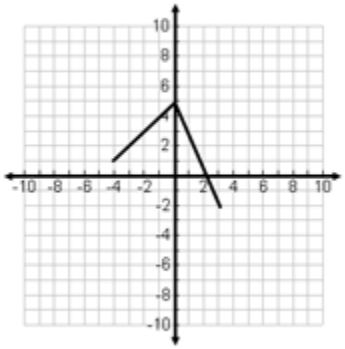
a) $-f(x)$



b) $f(x) + 1$



c) $f(x) - 5$



Polynomials:

- Vocabulary:
  - Leading Coefficient and Degree
  - Real vs. Complex Solutions
  - Zeros = x-intercepts = roots = solutions
  - y-intercept
  - end behavior
  - Standard and Factored Form
  - imaginary and complex numbers
- Skills:
  - Find values of polynomials by either direct or synthetic substitution
  - Add/Subtract/Multiply/Divide polynomials
    - Long or Synthetic Division
  - Factor polynomials:
    - Always start by looking for a GCF
    - Next, look for perfect squares
    - Lastly, use the box method or grouping where appropriate
    - Always re-check your factors to see if they can be factored further
  - Finding potential rational roots of polynomials and determining actual roots
  - Graphing Polynomials (without a calculator):
    - Finding relative min and max of a graph
    - From factored form or standard form
    - Understand end behavior and shape from degree and leading coefficient
    - In factored form: roots that touch vs. cross

12) Show all work as you perform the indicated operation:

a) $(4x^4 + 3x^3 - 2x + 9) + (5x^3 - 2x^2 - 3)$
b) $(8x^3 - 2x^2 + 6x) - (5x^3 - 4x^2 + 3)$
c) $(4x + 3)(3x - 2)$
d) $(5x + 6)^2$
e) $(x + 2)(4x^2 - 3x + 1)$
f) $(x - 1)(x^2 + 2x - 4)$

13) Factor the expressions completely:

a) $x^2 - 1$
b) $6x^2 - 17x + 5$
c) $x^4 + 8x^2 + 16$
d) $5x^4 - x^3 + 10x - 2$
e) $x^4 + 7x^2 + 12$
f) $x^4 - 2x^2 - 8$
g) $3x^2 - 18$
h) $x^3 - 2x^2 - 4x + 8$

14) Show all work as you solve the polynomials:

a) $x^3 + x^2 - 12x = 0$

b) $0 = 4x^3 - 8x^2 - 12x$

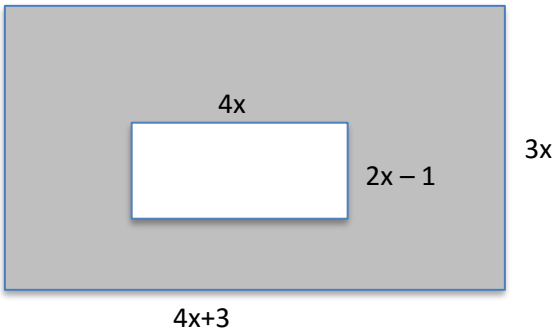
c) $0 = 20x^5 - 5x^3$

d) $3x^2 - 75 = 0$

e) $2x^2 + 72 = 0$

f) $x^4 - 5x^2 - 6 = 0$

15) Use the picture below to answer the questions:



a) Write a polynomial in <u>standard form</u> for the area of the larger rectangle.

b) Write a polynomial in <u>standard form</u> for the area of the smaller rectangle.

c) Write a polynomial in <u>standard form</u> for the area of the shaded region.

16) List the <u>possible/potential</u> rational roots for $f(x) = 3x^5 - 9x^3 + x - 6$

17) Find all the zeros of the function $f(x) = x^3 - 3x^2 - 6x + 8$.

18) Complete the end behavior for each without using a calculator:

a) $f(x) = -2x^4 + 6x + 8$

$as \;\; x \to \infty \qquad f(x) \to$ _____

$as \;\; x \to -\infty \qquad f(x) \to$ _____

b) $f(x) = 3x^5 - 6x^4 - 8x - 10$

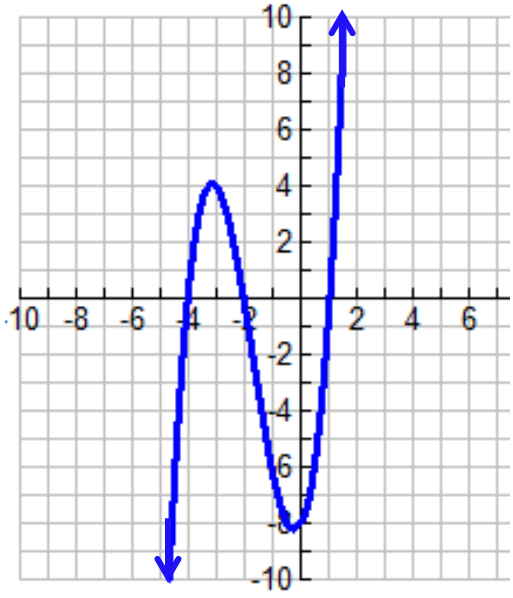
$as \;\; x \to \infty \qquad f(x) \to$ _____

$as \;\; x \to -\infty \qquad f(x) \to$ _____

19) The zeros of a function are x=1 and x=-2. The function does <u>not</u> pass through the x-axis at x = -2. Write one equation in factored form.

20) The zeros of a function are x=1, x=-2, and x=3. The function passes through the x-axis at all of these values. Write an equation in factored form.

21) Use the graph below to answer the following questions:



a) List the Zeros: _____

b) Write a possible equation for f(x):_____

c) Is the degree odd or even?

d) Is the leading coefficient positive or negative?

e) Estimate the relative maximum: _____

Estimate the relative minimum: _____

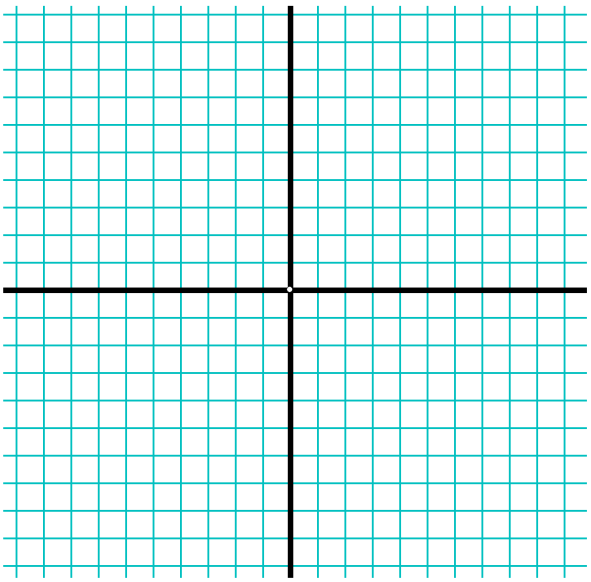
f) Describe the End Behavior:

as x→ -∞, f(x)→ _____

as x→ +∞, f(x) → _____

22) Graph the polynomial using your calculator to find the zeros and relative/local maximum and minimum. Sketch the graph using these points. Round all values to the nearest tenth.

$$f(x) = -(x-4)(x-6)(x-2)$$



Zeros: _____

Relative/local Maximum: _____

Relative/local Minimum: _____

Degree: _____

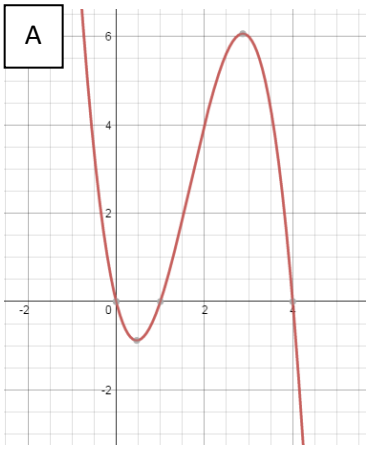
Leading Coefficient: _____

End Behavior: as x→ -∞, f(x)→ _____

as x→ +∞, f(x) → _____

23) Select which graph is: $f(x) = x(x-3)(x+1)$

A



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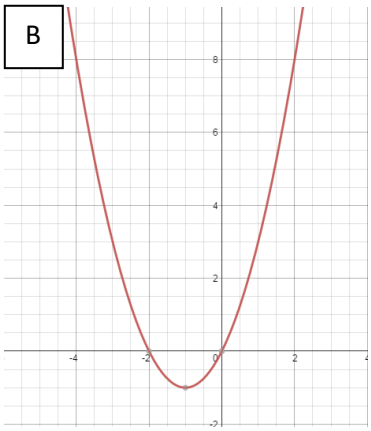

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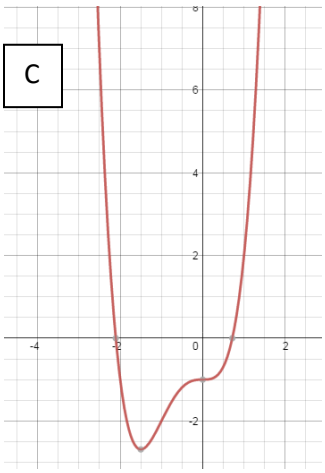

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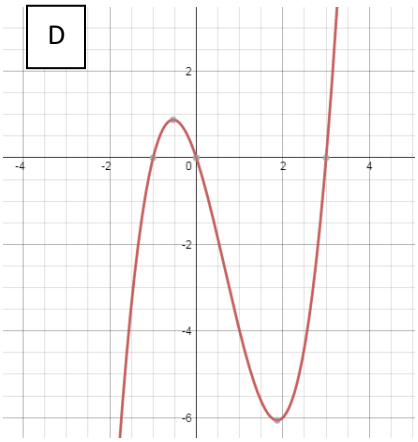


24) Factor each expression <u>completely</u>:

a) $x^3 - 6x^2 + 5x + 12$ given that *(x – 4)* is a factor

b) $x^3 - 4x^2 + x + 6$ given that *(x – 3)* is a factor

25) Perform the operations on complex numbers. Make sure to simplify your answer completely.

a) $i =$

b) $i^2 =$

c) $(5 - 8i) + (1 + 3i)$

d) $(2 + 3i) - (4 - 5i)$

e) $(4 + 5i)(2 - 3i)$

f) $(3 - i)(2 + i)$

g) Solve for x: $-2x^2 - 50 = 0$

26) Divide using long or synthetic division:

a) $(x^2 + 5x - 14) \div (x - 2)$

b) $(x^2 - 2x - 48) \div (x + 5)$

c) $(x^3 + x + 30) \div (x + 3)$

d) $(8x^3 + 5x^2 - 12x + 10) \div (x - 2)$

27) Find the remainder of: $(2x^3 - 4x + 5) \div (x + 4)$

28) More factoring!

a) $x^4 + 5x^2 - 36$

b) $x^3 + 2x^2 - 5x - 10$

c) $3x^2 + 10x + 8$

d) $3x^5 - 12x^4$

e) $3x^3 + x^2 - 12x - 4$

29) **Solve** each of the following for x. Round to 2 decimals if necessary.

a) $3x^3 - 12x = 0$

b) $5x^2 + 45 = 0$

c) $x^4 - 81 = 0$

d) $2x^2 - 3x - 9 = 0$

e) $x^3 - 7x^2 - 2x + 14 = 0$

f) $x^3 + 6x^2 - 9x - 54 = 0$

30) Write a function which would have the following characteristics:

a) Crossing the x axis at -2, 5, and touching the x axis at 1, with both ends of the graph moving towards positive infinity.

f(x)=_____

b) Crossing the x axis at 0 and touching the x axis at 4 with the left end of the graph moving towards positive infinity and the right end of the graph moving towards negative infinity.

f(x)=_____

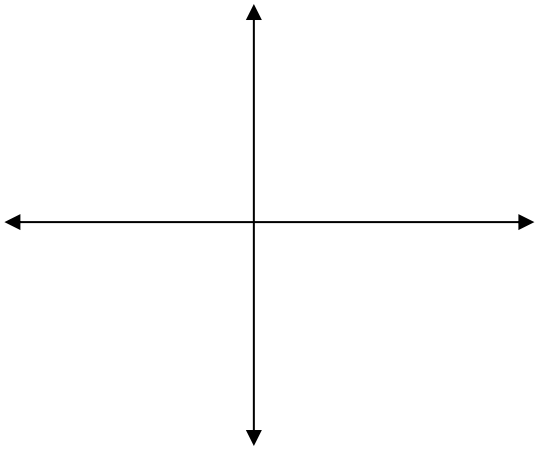
31) List all the possible rational roots of $2x^2 + 3x + 3 = 0$

32) Find all the zeros of the function $x^3 - 12x^2 + 35x - 24$

33) Is $(x + 5)$ a factor of $(x^3 + 4x^2 + 5x - 25)$? Show work to support your answer.

34) State the degree, the leading coefficient, and sketch the end behavior. Then sketch the graph by using that information and a table of values.

a) $y = x^3 + 3x^2 - 10x - 24$



Identify the degree: _____

y-intercept: _____

Real solutions: _____

How many complex solutions does the equation have?

_____

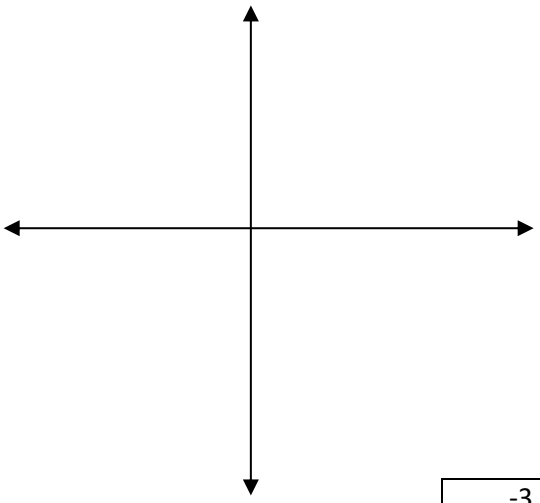
End behavior:

$as\ x \to \infty \quad f(x) \to$ _____

$as\ x \to -\infty \quad f(x) \to$ _____

-3	-2	-1	0	3

b) $y = -x^4 - 3x^3 - 7x^2 - 15x - 10$



Identify the degree: _____

y-intercept: _____

Real solutions: _____

How many complex solutions does the equation have?

_____

End behavior:

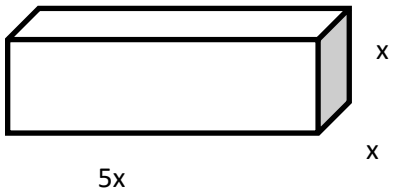
$as\ x \to \infty \quad f(x) \to$ _____

$as\ x \to -\infty \quad f(x) \to$ _____

-3	-2	-1	0

35) Find the real and complex solutions of $g(x) = x^4 - 81$

36) The volume of the prism is 135 in^3. Find the value of x <u>using algebra</u>. Show your equation and all work.



37) The volume *V* (in cubic feet) of a shipping box is modeled by the polynomial function $V(x) = x^3 - 2x^2 - 19x + 20$, where *x* is the length of the box.

a) What is the volume of the shipping box if the length is 10 feet?

b) Explain how you know $x = -2$ is *not* a possible rational zero.

c) Show that $x + 4$ is a factor of $V(x)$. Then factor $V(x)$ completely.

Exponents and Radicals:

- Skills:
  - Switch between radical and rational exponent form
  - Use properties of exponents and radicals to simplify expressions
  - Solve radical equations
    - Check for extraneous solutions
    - Know when solutions are ±

38) Write in radical form:

a) $x^{3/7}$

b) $x^{1/3}$

c) $(x^2)^{1/3}$

39) Write in exponential form:

a) $\sqrt[3]{x^2}$

b) $\sqrt{x^5}$

c) $\sqrt[4]{x}$

40) Simplify each expression completely without using a calculator:

a) $x^5 \cdot x^2$

b) $\frac{x^8}{x^2}$

c) $(y^3)^4$

d) $5^{3/4} \cdot 5^{7/4}$

e) $\left(3^{3/4}\right)^{1/3}$

f) $\left(\frac{x^2}{y^{-1}}\right)^{-1}$

g) $\sqrt[3]{\frac{64x^9}{8x^2}}$

h) $(a^2)^{\frac{3}{2}}$

i) $x^{\frac{1}{3}} \cdot x^{\frac{2}{5}}$

41) Evaluate without using a calculator:

a) $(9)^{3/2}$

b) $\frac{3^5}{3^2}$

c) $8^{-2/3}$

d) $\left(\frac{1}{64}\right)^{-2/3}$

e) $\left(\sqrt[3]{27}\right)^2$

f) $\sqrt[3]{24}$

g) $\left(\frac{1}{25}\right)^{-3/2}$

h) 16^0

42) Solve algebraically and check for extraneous solutions:

a) $\sqrt{8x+1} = 4 - x$

b) $\sqrt{x+6} = x$

c) $2\sqrt[4]{x-3} = 4$

d) $\sqrt[3]{2x+4} + 6 = 10$

e) $x^3 = 8$

f) $x^3 = -27$

g) $x^4 = 81$

h) $x^4 - 2 = 254$

43) Show all work as you solve each radical equation algebraically or graphically.

a) $\sqrt{x+25} = 4$

b) $\sqrt[3]{x} - 9 = -1$

c) $2\sqrt[3]{x-3} = 4$

d) $x + 1 = \sqrt{7x+15}$

44) The number of eggs, *E*, sold in a supermarket can be modeled by the equation $E = 4500\sqrt{0.5x+2}$ where *x* is the number of days past since Monday. On how many days past Monday will 9000 eggs be sold?