

Introduction to Calculus 50

Summer Work

- The following topics represent expected knowledge for Introduction to Calculus 50.
- Complete the problems on separate sheet paper.
- Access your earlier course work notes and online help when needed.
- Due the first day of your class meeting

COMPLEX FRACTIONS

Simplify each of the following complex fractions.

$$1. \frac{2 + \frac{2}{x}}{3 - \frac{3}{x^2}}$$

$$2. \frac{x - \frac{1}{3}}{3 - \frac{1}{x}}$$

$$3. \frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$$

$$4. \frac{\frac{1}{3} + \frac{1}{3x}}{\frac{1}{x} + \frac{1}{3}}$$

$$5. \frac{\frac{2}{x} - 2}{\frac{1-x}{x}}$$

$$6. \frac{\frac{1}{4x^2} - 1}{2 - \frac{1}{x}}$$

$$7. \frac{2 + \frac{4}{x-2}}{\frac{2}{x-2}}$$

$$8. \frac{\frac{3}{x} - \frac{4}{x^2}}{\frac{2}{x^3} + \frac{5}{x^2}}$$

FACTORING

Factor each of the following completely:

$$1. 3x^2 + 21xy - 54y^2$$

$$9. x^4 - 1$$

$$16. 6x^3 + 10x^2 + 3x + 5$$

$$2. 3x^2 + 24x + 48$$

$$10. -24x^3y^3 + 16x^4y^2 -$$

$$17. 1000 + 27a^3$$

$$3. 3m^2 - 18m + 27$$

$$8x^5y^3$$

$$18. s^3 - 64$$

$$4. 8x^3 - 200x$$

$$11. x^3 + 4x + x^2 + 4$$

$$19. y^3 + 125$$

$$5. 3x^3 + 78x^2 + 507x$$

$$12. 2x^3 + x^2 + 8x + 4$$

$$20. 3a^3 - 81x^3$$

$$6. x^6 - 16x^4$$

$$13. 15x^3 + 5x^2 + 3x + 1$$

$$21. (2x + 3)^3 - y^3$$

$$7. 4x^3 - 8x^2 - 25x + 50$$

$$14. 20n^3 + 12n^2 + 25n + 15$$

$$22. r^3 + 8b^3$$

$$8. 8x^3 + 27$$

$$15. 9p^3 + 3p^2 + 15p + 5$$

PROPERTIES OF LOGARITHMS

Expand each logarithm completely.

- | | | | |
|-----------------------------|------------------------|---------------------------|----------------------------|
| 1) $\ln \frac{1}{\sqrt{t}}$ | 2) $\log_3 11x$ | 3) $\log_3 \sqrt[3]{x+1}$ | 4) $\log_4 \sqrt{3x}$ |
| 5) $\log_2 \frac{z}{17}$ | 6) $\ln \frac{5}{x-2}$ | 7) $\log_5 \sqrt{xy}$ | 8) $\ln \frac{x^2 y}{z^7}$ |

Condense each logarithm into one logarithm.

- | | | |
|---|---|---|
| 1) $\log_5 2 + \log_5 3 + \log_5 4$ | 2) $\log_2 48 - \frac{1}{3} \log_2 27$ | 3) $\frac{2}{3} \ln 8 - 2 \ln 5$ |
| 4) $2 \log_{10} \sqrt{x} + 3 \log_{10} x^{\frac{1}{3}}$ | 5) $\frac{1}{2} (\log M - \log N - \log P)$ | 6) $5(\log A + \log B) - 2 \log C$ |
| 7) $\log_8 \sqrt{80} - \log_8 \sqrt{5}$ | 8) $\frac{1}{2} \ln 25 + \ln 2$ | 9) $\log_2 5 + \log_2 (x^2 - 1) - \log_2 (x - 1)$ |

RATIONAL EXPRESSIONS

Simplify:

- | | | |
|-------------------------------------|--|------------------------------------|
| 1) $\frac{x^3 - 9x}{x^2 - 7x + 12}$ | 2) $\frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$ | 3) $\frac{9 - x^{-2}}{3 + x^{-1}}$ |
|-------------------------------------|--|------------------------------------|

Perform the indicated operation on the rational expression

- | | |
|--------------------------------------|---|
| 1. $\frac{5}{8} - \frac{3}{8x}$ | 2. $\frac{2}{4x+12} + \frac{7}{x+3}$ |
| 3. $\frac{7}{x+2} - \frac{4}{x-5}$ | 4. $\frac{3}{y} + \frac{2}{y^4}$ |
| 5. $\frac{5}{4x^2} + \frac{3}{2x}$ | 6. $\frac{2}{x-3} - \frac{1}{x+7}$ |
| 7. $\frac{7}{3x} - \frac{2}{5}$ | 8. $\frac{3}{2x+6} + \frac{4}{6x+18}$ |
| 9. $\frac{3}{x+2} + \frac{4}{x-7}$ | 10. $\frac{1}{y+3} + \frac{4}{y^2+4y+3}$ |
| 11. $\frac{2}{5x} - \frac{3}{10x^3}$ | 12. $\frac{2x+3}{5x-30} - \frac{3x+4}{x-6}$ |

RADICALS AND RATIONAL EXPONENTS

REVIEW: n^{th} Root: If $b^n = a$, then b is the n^{th} root of a . This is written $\sqrt[n]{a} = b$. n is called the **index** of the radical. a is called the **radicand**. Roots as Rational Exponents: The n^{th} root, $\sqrt[n]{a}$, can be written as an exponent $a^{\frac{1}{n}}$. $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ - Notice the placement of the m and n . The root index is the denominator and the exponent is the numerator.

Rationalize the denominator:

1) $\frac{2}{\sqrt{3} + \sqrt{2}}$

2) $\frac{2}{1 - \sqrt{5}}$

3) $\frac{1}{1 + \sqrt{3} - \sqrt{5}}$

Evaluate.

1) $(-27)^{\frac{1}{3}}$

2) $(-16)^{\frac{1}{4}}$

3) $16^{-\frac{3}{4}}$

Write in exponential form.

1) $\sqrt[5]{a^2}$

2) $(\sqrt[6]{a})^5$

Write as a radical expression with a smaller index.

1) $\sqrt[4]{49}$

2) $\sqrt[14]{x^7}$

Simplify.

1) $\sqrt[4]{x} \cdot \sqrt{x}$

2) $\frac{\sqrt[3]{x^2}}{\sqrt[4]{x}}$

3) $\sqrt[3]{2} \cdot \sqrt[3]{5}$

4) $\frac{(2a^2)^3}{b}$

5) $\sqrt{9ab^3}$

6) $\frac{a(2/b)^3}{3/a}$

7) $\frac{ab - a}{b^2 - b}$

8) $\frac{a^{-1}}{b^{-1}\sqrt{a}}$

9) $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{3/2}}{a^{1/2}}\right)$

LITERAL EQUATIONS

1. Solve for a.

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

2. Solve for v.

$$tv + tv = s$$

3. Solve for a.

$$V = 2(ab + bc + ca)$$

4. Solve for positive r.

$$A = 2\pi r^2 + 2\pi rh$$

5. Solve for d.

$$2x - 2yd = y + xd$$

6. Solve for x.

$$\frac{2x}{4\pi} + \frac{1-x}{2} = 0$$

LIBRARY OF FUNCTIONS AND END BEHAVIOR OF POLYNOMIALS

Describe the end behavior of the graphs of the following functions. Decide if $f(x) \rightarrow +/\infty$ as $x \rightarrow +/\infty$. You do NOT need to completely distribute the ones that are factored—all that matters is the term with the highest power of x .

	as $x \rightarrow \infty, f(x) \rightarrow ?$	as $x \rightarrow -\infty, f(x) \rightarrow ?$
1. $f(x) = -2x^3 + 6x - 11$		
2. $f(x) = x^4 - 5x^3 - x^2 + 2x + 1$		
3. $f(x) = 2(x-1)(x+3)(x-5)$		
4. $f(x) = -x^2(x+7)(x-3)$		
5. $f(x) = 3x^2(x-1)^3(x+2)$		

Graph each of the following parent functions WITHOUT a calculator:

6. $f(x) = x^2$

7. $f(x) = x^3$

8. $f(x) = \sqrt{x}$

9. $f(x) = \sqrt[3]{x}$

10. $f(x) = \frac{1}{x}$

11. $f(x) = \frac{1}{x^2}$

LINEAR EQUATIONS

Determine the equations of the following lines:

- 1) the line through $(-1, 3)$ and $(2, -4)$;
- 2) the line through $(0, 6)$ with slope -4 ;
- 3) the line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$;
- 4) the line through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$.

DOMAIN AND RANGE

Find the domain of the functions:

$$1) f(x) = \frac{3x-1}{\sqrt{x^2+x-2}}$$

$$2) f(x) = \frac{5x-3}{2x+1}$$

$$3) f(x) = 7$$

FUNCTIONS

Simplify: $\frac{f(x+h) - f(x)}{h}$, where:

$$1) f(x) = 2x + 3$$

$$2) f(x) = \frac{1}{x+1}$$

$$3) f(x) = x^2$$

INVERSE FUNCTIONS

Find the inverse of the function:

$$1) f(x) = 2x + 3$$

$$2) f(x) = \frac{x+2}{5x-1}$$

$$3) f(x) = x^2 + 2x + 1, x > 0$$

ASYMPTOTES

Determine the equations for the horizontal and vertical asymptotes:

$$1) f(x) = \frac{4x-2}{2x+4}$$

$$2) f(x) = \frac{2x-3}{x^2-7x+12}$$

$$3) f(x) = \frac{2x^2+3x-5}{x-1}$$