Intro to Calculus: Final Exam Review

Formulas Given:

Chain Rule: $\frac{d}{dx}\left[f\left(g(x)\right)\right] = f\,'\,(g(x))g'x$

Product Rule: $\frac{d}{dx}\left[f(x)\left(g(x)\right)\right]$ = f '(x)g(x) + g'(x)f(x)

Quotient Rule: $\frac{d}{dx}\,\frac{f(x)}{g(x)} = \frac{f'(x)g(x)-g'(x)f(x)}{[g(x)]^2}$

Position Function: $h(t) = -16t^2 + v_0t + h_0$

Revenue = xp

Profit = Revenue – Cost
P(x) = R(x) – C(x)

Exponentials & Logarithmic Derivatives

y = a^x	y' = ln a (a^x)
y = a^u	y' = ln a $(a^u)\ \frac{du}{dx}$
y = ln x	y' = $\frac{1}{x}$
y = ln u	y' = $\frac{1}{u} \cdot \frac{du}{dx}$
y = $\log_a x$	y' = $\frac{1}{\ln a} \cdot \frac{1}{x}$
y = $\log_a u$	y' = $\frac{1}{\ln a} \cdot \frac{1}{u} \cdot \frac{du}{dx}$

Exam Breakdown: Units of Study

- Limits & Continuity
- Derivatives
- Velocity & Marginal Functions
- Curve Sketching
- Optimization
- Exponential & Logarithmic Functions
- Integrals

Non-Calculator: 106 Pts

Multiple Choice: 38 pts

Open Ended: 68 Pts

Calculator: 28 Points

Total: 134 Points

Unit 1: Limits & Continuity

1. Find $f(x + \Delta x)$ if $f(x) = x^2 + 4$

2. Determine the $\lim_{x\to 2} 3x^2 + 5$

3. Determine the $\lim_{x\to -1} \frac{x^2+3x+2}{x^2+1}$

4. Determine the limit: $\lim_{x\to 3^-} \sqrt{9 - x^2}$

5. Determine the limit: $\lim_{x\to 2} \frac{x}{x-2}$

6. Determine the limit: $\lim_{x\to 1} f(x) = \begin{cases} x^2 + 4, x \neq 1 \\ 2, \quad x = 1 \end{cases}$

7. Determine the limit: $\lim_{x\to -1} \frac{x^2-5x-6}{x+1}$

8. Find: $\lim_{x\to 1} f(x) = \begin{cases} 4 - x, & x < 1 \\ 4x - x^2, & x > 1 \end{cases}$

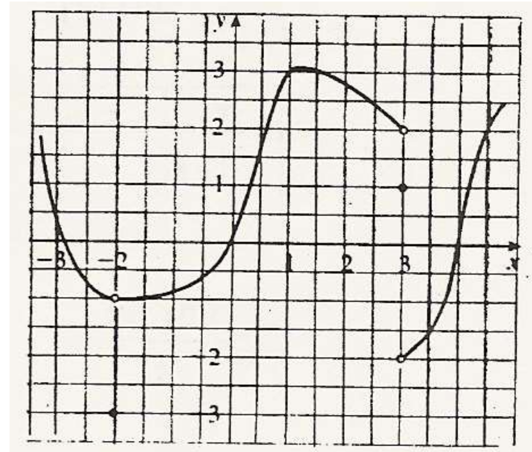
9. Find $\lim_{x\to 0} f(x) = \begin{cases} 3 + x^2, & x > 0 \\ x + 1, & x \leq 0 \end{cases}$

10. Find the $\lim_{x\to 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x}$

11. Find $\lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ when $f(x) = 3x^2 + 4x - 1$

12. Find $\lim_{x\to -1} (3x - 5)$

13. Determine the following given the graph f(x) :



a) $\lim_{x \to 3^+} f(x) =$

b) $\lim_{x \to 3^-} f(x) =$

c) $\lim_{x \to 3} f(x) =$

d) f(3) =

14. Find the average rate of change for the function f(x) = $2x^2 - 3$ over the interval [-2,4]

15. Determine the intervals in which the function is continuous: f(x) = $\frac{(x+1)(x-3)}{x-2}$. Determine if the discontinuities are removable or non-removable.

16. Let f(x) = $\frac{5}{x-1}$ and g(x) = x^4.

a. Determine the continuity of f(x)

b. Determine the continuity of g(x).

17. Determine the discontinuities and types of the function.

a. f(x) = $\begin{cases} x^2 + 1, x \geq 3 \\ 4x + 2, x < 3 \end{cases}$

b. f(x) = $\frac{x-4}{x^2 + x - 2}$

c. f(x) = $\frac{x+3}{x^2 + 2x - 3}$

18. Find the value of **a** such that f(x) is continuous $f(x) = \begin{cases} ax + 3, & x \leq 2 \\ 4x + a, & x > 2 \end{cases}$

Unit 2: Derivatives

Find the derivative of the given functions.

19. $f(x) = 3x^5 - 4x^2 + 5$

20. $f(x) = 15$

21. $f(x) = 4(x - 5)$

22. $f(x) = (3x - 5)^2$

23. $f(x) = \dfrac{5}{(x+3)^2}$

24. $f(x) = \dfrac{4}{x}$, $f^{(5)}(x) = ?$

25. $f(x) = \dfrac{x}{5x - 2}$

26. $f(x) = (3x - 4)^2 (x - 6)^4$

27. f(x) = $\dfrac{x^2 - 4x}{\sqrt{x}}$

28. f(x) = $4\sqrt[3]{x}(2x+3)$

29. f(x) = $\dfrac{2x^3}{7}$

30. f(x) = $\dfrac{2x^2 - 2x + 9}{\sqrt{x}}$

31. f(x) = $\dfrac{3x}{x+1}$

32. f(x) = $(2x+5)(x^2 - 2x + 1)$

33. f(x) = $\dfrac{x}{\sqrt{x-1}}$

34. f(x) = $\sqrt{4 - 3x + x^2}$

35. f(x) = $x^3\sqrt{x+1}$

36. f(x) = $-8(1 - x)^2 + 7(1 - x) + 2$

37. $y = \dfrac{x^2}{3x^2 - 1}$.

38. f(x) = (3x – 4)(x² – 2x + 1).

39. $f(x) = \left(2x^2 + 5\right)^7$

40. $y = \sqrt[3]{x^2 + x}$.

41. $f(x) = (x^3 + 3x^2 - 6x + 1)$

42. Determine y''' of $y = 4x^5 + 2x^{-1}$.

Find $\frac{dy}{dx}$ by using implicit differentiation.

43. $y^2 - 3xy + x^2 = 7$.

44. $x^3 - y^2 = 5$

45. $\dfrac{x}{(y-4)} = 5$

46. $x^2y + y^2 = x$

47. $5x^2 - 3xy + 7 = 2$

48. $x^2 + y^2 = 2xy$

49. $y - x = 5 - xy$

50. Use implicit differentiation to find the equation of the tangent line for the graph $2x^2 + 3y^2 = 11$ at the point: (2,1)

51. Determine the equation of the tangent line to the graph of $x^2 + 2y^2 = 3$ at the point (1, 1).

52. Determine the slope of the equation $f(x) = \dfrac{2}{(3x-1)^3}$ at the pt. (1, ¼).

53. Determine the slope of the tangent line to the graph of $f(x) = -x + 3$ when $x = 2$.

54. Determine the equation of the tangent line for the following functions.

a. $f(x) = 3x^2 - 10x$ (2, -8)

b. $x^2 + 3y = 5$ (2, 1/3)

55. Determine the equation of the line tangent to $f(x) = 2x^2 - 4$ at the point (3, 14).

56. Determine the value of the derivative of $f(x)=1+\sqrt{x}$ at the point (9, 4).

57. Determine the value of the derivative of $f(x)=\dfrac{x+2}{x+1}$ at the point $\left(1,\dfrac{3}{2}\right)$.

58. Determine the point(s) on the graph of the function $f(x)=x^3-2$ where the slope is 3.

Unit 3: Velocity & Marginal Functions

59. The velocity function for an object is given by $s'(t)=-10t^2+4$, where s is measured in feet and t is measured in seconds. What is the instantaneous velocity when t = 2?

60. The height s (in feet) of an object fired straight up from ground level with an initial velocity of 150 feet per second, is given by $s = -16t^2 + 150t$, where t is the time in seconds. How fast is the object moving after 5 seconds?

61. A manufacturer determines that the profit derived from selling x units of a certain item is given by $P = -0.05x^2 + 200x - 20$. Determine the marginal profit for a production level of 50 units.

62. Suppose the position equation for a moving object is given by $s(t) = -3t^2 + 2t + 5$, where s is measured in meters and t is measured in seconds.

a. Determine the maximum height.

b. Determine the time it takes to hit the ground.

c. Determine the velocity at 2 sec.

d. Determine the velocity when the object hits the ground.

e. Determine the acceleration when the object hits the ground.

63. Estimate the additional cost of the cost function $C(x) = 2.4\sqrt{(x)} + 400$ if the production level is increased from 50 to 51 units.

64. The demand function for a particular commodity is given by $p = 600 - 3x$.

a. Determine the revenue equation.

b. Determine the marginal revenue equation.

c. Determine the marginal revenue when $x = 30$.

65. A manufacturer determines that the profit derived from selling x units of a certain item is given by $P = -x^2 + 101x$.

a. Determine the marginal profit for a production level of 50 units.

b. Explain the meaning of your answer from part a.

66. An assembly plant produces bushings to install in one of its products. The estimated cost per busing over the next five years is given by $C = 4(3.75t + 3)^{1.5}$, where t is time in years. Determine the rate of change of cost for the fifth year.

67. An object is propelled straight up from ground level with an initial velocity of 310 ft/ sec.

a. Determine the position function of the object.

b. Determine the velocity function.

c. Determine the acceleration function.

d. Find the time it takes for the object to hit the ground.

e. Find the velocity when the object hits the ground.

f. Find its height after 8 seconds.

Unit 4: Curve Sketching

68. Determine all critical numbers of $f(x) = \dfrac{x-1}{x+3}$.

69. Determine all critical numbers of $f(x) = x^3 - 3x^2$.

70. Determine the open intervals on which the function $f(x) = \sqrt[3]{x^2 - 1}$ is decreasing.

71. Determine all relative extrema of $f(x)=3x^5-25x^3+60x+2$.

72. Given that $f(x)=\dfrac{-4}{x+2}$, determine the intervals where y is concave up or concave down and any inflection points.

73. Determine all points of inflection of the function $f(x)=x^4-x^3$.

74. Determine all relative extrema of $f(x) = 3x^5 - 25x^3 + 60x + 2$.

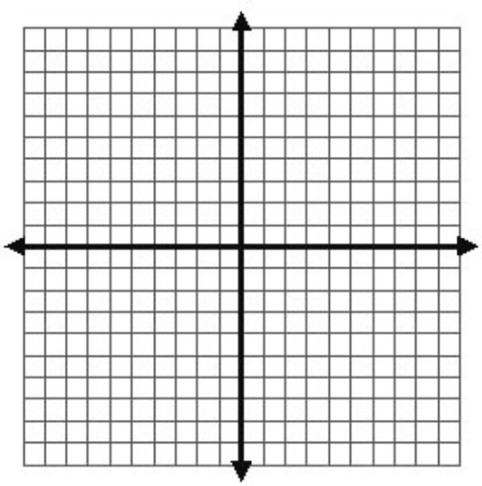
For questions 75-77, use the function: $f(x) = x^3 - 9x^2 + 24x - 10$

75. Determine the critical value(s).

76. Determine the interval(s) of increasing/decreasing.

77. a. Sketch a graph of the function

b. Determine the (x,y) coordinates of any relative and/or absolute extrema. If none, indicate in the space provided.



b. Relative maximum(s):__________________

Relative minimum(s):__________________

Absolute maximum(s):__________________

Absolute minimum(s):__________________

Unit 5: Optimization

78. Determine two positive integers whose sum is a minimum if the product of the two numbers is 36.

79. An open box with a rectangular base is to be constructed from a 16” by 21” piece of cardboard by cutting out squares from each corner and bending up the sides. Find the dimensions of the box that will have the largest volume.

80. Mrs. Freehill is fencing in her dog. It has an area 500 feet^2. One side is bordered by the house and requires no fence. The side parallel to the house is to be made of wood and costs $10/ft. The other sides are to be made of plastic and cost $25/foot. What dimensions would minimize the cost? What is the cost?

81. If the price charged for a candy bar is p(x) cents, then x thousand candy bars will be sold in a certain city where $p(x) = 125 - \frac{x}{14}$. How many candy bars must be sold to **maximize revenue**?

    (Recall: $R = xp$)

Unit 6: Exponential & Logarithmic Functions

82. Determine y' if $y=e^{1/x}$.

83. Simplify $e^{\ln(3x+1)}$.

84. A deposit of $1000 is made into a fund with an annual interest rate of 9%. Determine the time required for the investment to double if the interest is compounded continuously. ($A = Pe^{rt}$)

85. Determine $\frac{dy}{dx}$ for $y=\ln(5-x)^6$.

86. Determine f '(x) if $f(x)=x^2 \ln x$.

87. A certain type of bacteria increases continuously at a rate proportional to the number present. If there are 500 present at a given time and a 1000 present 2 hours later, how many will there be 5 hours from the initial time given?

Unit 7: Integrals

Determine the indefinite integral.

88. $\int (2x^4 + 3x^2 - 2x)dx$.

89. $\int \frac{3x}{(x^2-7)^4} dx$

90. $\int \frac{x}{(x^2-1)} dx$

91. $\int \frac{e^{3/x}}{x^2} dx$

92. $\int (2x^3)(x^4 - 2)dx$

93. $\int \frac{3x^5 - 5x^3}{x^4} dx$

94. $\int \frac{3}{x^2} dx$.

95. $\int \frac{3y}{\sqrt{y^2+1}} dy$.

96. $\int xe^{5x^2} dx$.

97. $\int \frac{x^2}{1-x^3} dx$.

Evaluate the definite integral.

98. $\int_{-3}^{0} (4x^4 - 5x^3 + 8)dx$.

99. Evaluate $\int_{-1}^{1} 4x(2x^2 + 3)dx$

100. Determine the area of the region bounded by the graphs of $y = -x^2 + 2x$ and y = 0.

101. Determine the area of the region bounded by the graphs of $y = x^2 + 1$ and y = -x + 3.

102. Determine the area of the region bounded by the graphs of $y = \sqrt{x+4}$, $y = \frac{1}{x}$, x = 3 and x = 6.

103. Determine the area of the region bounded by the graphs of $y = 4x - 1$, $y = -x + 3$, x = 3 and x = 6.