Pre-Calculus 40 Final Review Guide Name:_____

First Semester Topics:

NONCALCULATOR:

1. Determine if the following equations are polynomials. If they are polynomials, determine the degree, leading coefficient and constant term.

a. $f(x) = 3x$ b. $g(x) = 4x^5 - 3x^2 + 2$ c. $h(x) = \sqrt{5x^3 - 2x^2 + 12}$

2. Determine whether the x value is a solution to the equation $0 = x^4 + 6x^3 + 9x^2 + 24x + 20$

a. x = 2 b. x = -1 c. x = 0

3. Determine all real solutions of $2x^3 - x^2 - 7x + 6 = 0$. NONCALCULATOR! Show Factored Form.

4. Find all the x-intercept(s) of f(x) = $x^3 + 3x^2 - 4x - 12 = 0$. NONCALCULATOR!

5. Simplify the Rational Expressions:

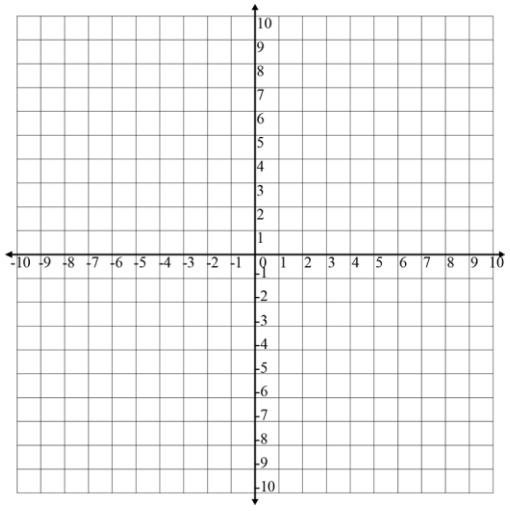
a. $\frac{3p^2-9p-30}{6p^2+6p-12}$ b. $\frac{x^2+8x+15}{2x^2-18}$ c. $\frac{3x^2-4x-4}{x^2-4}$

6. Determine the vertical asymptote(s), horizontal asymptote and domain of the functions.

a. $f(x) = \frac{x^2-4}{x^2-2x-8}$ b. $h(x) = \frac{x+4}{x^2+x-12}$

7. Identify the asymptotes/holes and any other critical points on the function below. Then graph on the axis provided.

$g(x) = \frac{5x+1}{x^2-3x}$



8. Create a rational function with the following characteristics:

Vertical asymptote x = 3 and x= 2

Horizontal asymptote y =3

X-intercepts (0,0) and $(-\frac{1}{3}, 0)$

9. Evaluate (non-calculator):

a. $\log_3 81$ b. $\log_2 \frac{1}{16}$ c. $\ln e^{-4}$

10. Solve the equation (non-calculator):

a. $32^{x+1} = 8^{5x}$ b. $16^{2x+1} = 64^{3x+2}$ c. $\log_2 4 + \log_2(x-1) = \log_2(x+1)$

d. $\ln(3x+1)+2\ln(3)=\ln(x^2+19x)$ e. $\log_2 16 = x+3$ f. $\log_3 27 = 2x+1$

g. $log_4(3x+4)=3$ $h. 5^x = 125$ $i.\ 3^{x+1}=81$

11. Simplify the expression into a+bi form:

a. $(5+\mathrm{i})(3-2\mathrm{i})$ b. $\frac{6+2i}{3-2i}$ c. $\frac{1-5i}{4+i}$ d. $(2+3i)-(4-2i)$

12. Given $f(x)=x^2-2x+3\ and\ g(x)=x+6$. Determine:

- f(3)
- (f∘g)(2)
- (g∘f)(3)
- (f+g)(4)
- (f-g)(-5)

13. Find the inverse function given $f(x)=\frac{x+1}{2x}$

14. Verify by composition that f and g are inverses.

$f(x)=\sqrt{x-1}\ and\ g(x)=x^2+1$ b. $f(x)=2x+1\quad and\ g(x)=\frac{x-1}{2}$

CALCULATOR:

15. Paula invests $250 at 3.75% compounded monthly. Determine the amount in her account after 15 years.

16. Frederico invests $100 at 3.75% compounded continuously. How long will it take for him to double his investment?

17. What principal invested at 4.65% continuously for 10 years will yield $12,000? Round to 2 decimal places.

18. The decay of a 50 mg sample of radium is given by the equation $R(t) = 50e^{-0.000433t}$, where t is time in years and R(t) is the amount of radium at a given point in time.

a. How long will it take for the sample to decay to 10 mg?

b. What is the half life of radium?

19. The number of articles making up an on-line open-content encyclopedia increased exponentially during the first few years. The number of articles, A(t), t years after 2001, can be modeled by $A(t) = 16{,}198(2.13)^t$.

a. According to this model, how many articles made up the encyclopedia in 2001?

b. At what rate is the number of articles increasing?

c. During which year did the encyclopedia reach one million articles?

d. Predict the number of articles there will be at the beginning of 2018.

Second Semester Topics:

NONCALCULATOR:

20. Write each degree measure in radians as a multiple of π and each radian measure in degrees.

a. $136°$ b. $-45°$ c. $\frac{3\pi}{4}$ d. $-\frac{5\pi}{6}$

21. Find the exact value of each trigonometric function, if defined. If not defined, write undefined.

a. $\tan(-45°)$ b. $\cos\left(\frac{3\pi}{2}\right)$ c. $\csc\left(\frac{5\pi}{6}\right)$ d. $\sin(900°)$ e. $\sec\left(\frac{\pi}{2}\right)$

22. Determine all solutions to the equation on $[0,2\pi)$

a. $4\tan(x) - 7 = 3\tan(x) - 6$

b. $9 + \sin^2(x) = 10$

c. $7\cos(x) = 5\cos(x) + \sqrt{3}$

d. $5\sin(x) + 2 = \sin(x)$

23. Find the exact value of each of the following WITHOUT using a calculator!

a. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =$ b. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ c. $\arccos\left(-\frac{\sqrt{2}}{2}\right) =$ d. $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right) =$

e. $\sin^{-1}(\sin(-\pi)) =$ f. $\cos\left(\arctan\left(\frac{5}{2}\right)\right) =$ g. $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right) =$ h. $\cos\left(sin^{-1}\left(\frac{3}{8}\right)\right) =$

24. Find the exact value of each trigonometric function.

a. $\cos 15°$ b. $\sin \frac{19\pi}{12}$ c. $\tan 255°$

d. $\cos 25° \cos 35° - \sin 25° \sin 35°$ e. $\sin 135° + \tan 60°$ f. $\cos(-120°) - \sin(315°)$

25. Establish/Prove the identities.

a) *i.* $(sec^2 x - 1)cos^2 x = sin^2 x$ *ii.* $sinx(cscx - sinx) = cos^2 x$

Simplify.

b) $sec^2 x(1 - cos^2 x)$

26. Let $\overrightarrow{AB}$ be the vector with initial point *A*(10, –4) and terminal point *B*(–1, –3). Write $\overrightarrow{AB}$ as a linear combination of the vectors i and j.

27. Find the component form of $\overrightarrow{AB}$ with initial point *A*(–12, 7) and terminal point *B*(8, –2).

28. Given **r** = $\langle 3, 9 \rangle$ and **s** =$\langle -3, 6 \rangle$

a. 2r – s b. 5**r** – 2**s** c. r + 2s

29. Write the following parametric equations in rectangular form:

a. $x = t - 1,\ y = 2t^2 + 6$ b. $x = 4\cos\theta \quad y = 2\sin\theta$

c. x = 3t + 9, y = $t^2 - 7$ $d.\ x = t^2 + 1,\ y = 4t + 3$

30. Find the component form of $\overrightarrow{AB}$ given

a. $|v| = 12$ and direction angle $\theta = \frac{5\pi}{3}$

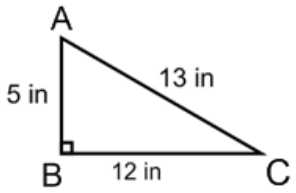
b. $|v| = 5$ and direction angle $\theta = 120°$

31. Given the matrices below, what is 2A + 3B? (No Calculator)

A = $\begin{bmatrix} 5 & 0 & 9 \\ -1 & 4 & -6 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 1 & -4 \\ 0 & 3 & -9 \end{bmatrix}$

CALCULATOR:

32. In ΔABC below, find the following values.



a. Angle *A* = _____

b. Angle *C* = _____

c. *tan A* = _____

33. A pilot needs to begin his descent when his plane is 7.5 km above ground and 200 km straight to the airport. At what angle should his decent be so that he can fly in a straight line from the point of initial decent to the ground? How much ground will he pass from the point of initial descent until he touches down at the airport?

34. If a building is 423 ft tall and the angle from its shadow to the top of the building is 43°, determine the length of the shadow.

35. A blimp was flying above Fairfield the other day at an altitude of 425 meters. Emily was in the blimp and she saw the high school. She calculated the angle of depression from the blimp to the entrance of the high school was about 48°. If she dropped a rock out of the blimp and the rock fell straight to the ground, how far away from the high school would the rock land? Round your answer to the nearest meter.

36. Given a triangle with the following dimensions, solve for the remaining sides and angles.

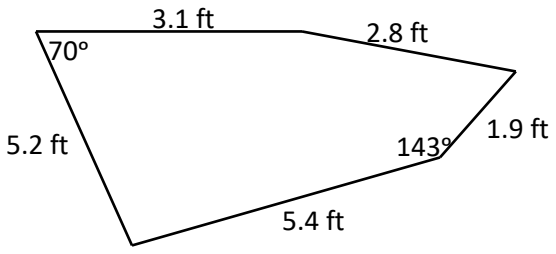
a. $a = 11cm, b = 6\ cm, A = 22^o$ b. $a = 13\ m, b = 12\ m, c = 8m$ c. $a = 9\ cm, b = 10\ cm, C = 42^o$

d. $a = 5\ cm, A = 36^o, B = 42^o$ e. $A = 63^o, a = 18in, b = 25in$ f. $A = 20^o, a = 4mm, b = 6mm$

37. Determine the area of each triangle to the nearest tenth.

a. $A = 95°,\ b = 12m, c = 18\ m$ b. $a = 44, b = 47, c = 53$

38. Mrs. Shannon wants a uniquely shaped blanket for the hours she will be lounging at the beach this summer. If the blanket is in the shape below determine the size of the blanket.



39. A plane takes off at 220 miles per hour at an angle of 51° with the ground. Find the magnitude of the horizontal and vertical components of its velocity. Round to the nearest tenth.

40. Charles leaves his apartment and walks 55° east of north for 1000 feet and then walks 300 feet due north to go bowling. Write a vector to represent each stage of Charles' trip. How far and at what quadrant bearing is Charles from his apartment when he arrives at the bowling alley?

41. Determine the direction angle and magnitude of the following vectors.

a. $\langle 2, -2 \rangle$ b. $\langle 0, 7 \rangle$ c. $\langle -2, 5 \rangle$

42. Find the component form of $\overrightarrow{AB}$ given $|v| = 12$ and direction angle $\theta = 120°$.

43. Charles is pulling a wagon with a force of 315 Newtons at angle of 37° with the ground. Draw a diagram showing the vertical and horizontal components and then find the magnitudes of horizontal and vertical components of the force. Round to the nearest tenth.

Horizontal Component: _____

Vertical Component: _____

44. Suppose Mr. Ebling kicks a soccer ball with an initial velocity of 150 feet per second at an angle of 30° to the horizontal. Round all answers to the *nearest tenth.*

a. Write a set of parametric equations that describe the position of the ball as a function of time.

b. How high is the ball after 1 second?

c. How long is the golf ball in the air?

d. When is the ball at its maximum height?

e. What is the maximum height of the golf ball?

f. How far away did the golf ball land?

45. Determine AB for the matrices below. Use of a calculator is permitted.

$$A = \begin{bmatrix} 6 & 9 & 1 \\ 4 & 0 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -2 \\ 4 & 3 \\ 5 & -5 \end{bmatrix}$$

46. Write a matrix for the system of equations. Then solve for each variable using your calculator. Round to two decimal places.

$2x + 7y - 2z = 5$
$3y - x = 6z + 12$
$-3z = 2x + 14 - 4y$