The following is extra practice for the final exam. The final exam will cover all material covered throughout the year. Note: You should STUDY in addition to completing this packet!!!

Exam Format:

Non-calc: 22 multiple choice and 15 open ended Calc: 5 multiple choice and 7 open ended

NON-CALCULATOR PRACTICE:

Matrices and Complex

1. Evaluate each of the following using matrices A, B and C

$A = \begin{bmatrix} -1 & 5 \\ 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 2 & -1 \\ 0 & -5 & 3 \end{bmatrix}$ $C = \begin{bmatrix} -1 & 0 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

a. $A + C$ b. $2B + C$ c. $B - \frac{1}{2}C$

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2. Solve the for the missing variables in the matrix equation below.

$$\begin{bmatrix} 3 & x & 5 \\ -2 & 1 & 4 \end{bmatrix} - 3\begin{bmatrix} -1 & 2 & 4 \\ y & -2 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & -7 \\ 5 & \frac{1}{2}z & 4 \end{bmatrix}$$

$x =$ ____, $y =$ _____, $z =$ ______

3. Simplify the following expressions: Recall a complex number in simplified form is of the form $a + bi$

a. $3i^2 + 4i^3 - 8i + 4$ b. $\frac{3-2i}{4+3i}$ c. $4(2-i)^2 - (3+i)$ d. $\frac{-1+i}{3-i}$

Odd/Even/Symmetry

4. Determine if the equations have x-axis, y-axis, origin or no symmetry. Then classify whether the function is even, odd, or neither.

a. $y = x^6 - 4x^2$ b. $y = 3x^5 + 2x - 10$ c. $f(x) = \frac{x^3}{x^2-7}$ d. $f(x) = \frac{x}{x^5-x}$

Polynomials and Rationals

5. Describe the end behavior of each function using limit notation.

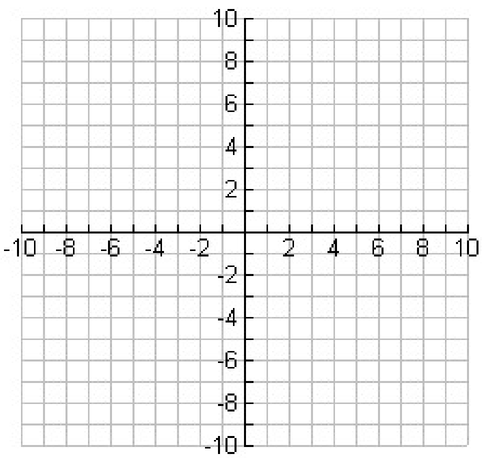
a. $h(x) = -4x^6 - 2x^2 + 1$

b. $h(x) = x^7(5 - x)(2x + 3)^3$

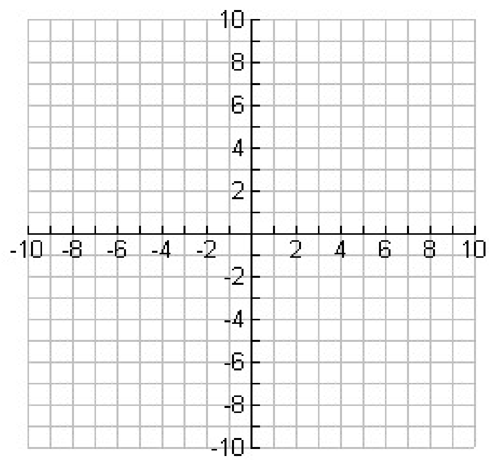
6. For the following functions determine the following:

i. Determine the end behavior of the graph.
ii. Determine the zeros and state the multiplicity of any repeated zeros
iii. Use this information to sketch a graph of the function.

a. $g(x) = -3x^2(x - 3)^4(x + 2)$



b. $f(x) = 2x^3(x + 3)^5 - 6x^4(x + 3)^4$



7. For the following functions,
a) Find the possible rational zeros of *h(x).*
b) Find ALL the zeros of *h(x)*.
c) Write h(x) as the product of linear and irreducible quadratic factors.
d) Write *h(x)* as the product of linear factors.

a. $h(x) = 2x^4 - x^3 - 17x^2 + 7x + 21$

b. $f(x) = x^4 + 2x^2 + 8x + 5$

c. $x^4 - x^3 + 7x^2 - 9x - 18 = 0$

d. $x^3 - 7x^2 + 16x = 10$

8. Write a polynomial with least degree that has the following zeros. $-3, 1, 4i$

9. Is $(x-2)$ a factor of $x^3 + 3x^2 + 5x - 30$? If yes, factor the polynomial completely.

10. Solve the inequalities.

a. $6x^3 + 10x^2 + 15x < -25$

b. $(x-4)^2 + 1 \geq 0$

c. $(x-4)(3x+6)^3(x+7)^3 > 0$

11. Determine the domain.

a. $f(x) = \frac{x^2+5x+6}{x+7}$

b. $g(x) = \frac{x+3}{x^2+2x-3}$

c. $f(x) = \frac{2x^3+7x^2-4x}{x^3+2x^2-3x}$

12. Analyze the graphs of the following rational functions: Include domain, discontinuities, end behavior, asymptotes and intercepts. Then use this analysis to sketch the function.

a. $f(x) = \frac{3x}{x-2}$

Domain:

Discontinuities:
Hole (removable):

VA (infinite):

End Behavior:

HA:

OA:

x-intercept:
y-intercept:

b. $f(x) = \frac{x-2}{x^2-6x+8}$

Domain:

Discontinuities:
Hole (removable):

VA (infinite):

End Behavior:

HA:

OA:

x-intercept:
y-intercept:

c. $f(x) = \frac{x^2+2x-3}{x+2}$

Domain:

Discontinuities:
Hole (removable):

VA (infinite):

End Behavior:

HA:

OA:

x-intercept:
y-intercept:

13. Simplify the following rational expressions:

a. $\frac{2x^2-5x-3}{x^2-9}$ b. $\frac{4}{x-3}-\frac{2}{x+2}$ c. $\frac{\frac{5}{x-2}}{\frac{x}{x+1}+3}$ d. $\frac{\frac{2}{x}+\frac{x}{x-2}}{1-\frac{3}{x^2-2x}}$

14. Solve the equation.

a. $\frac{1}{x}+\frac{4x+12}{x^2-3x}=\frac{2}{x-3}$ b. $\frac{1}{x-2}+\frac{1}{x^2-7x+10}=\frac{6}{x-5}$

15. Solve the inequalities:

a. $\frac{(x+3)}{(x-5)}\leq 0$ b. $\frac{7}{x+3}>1$

Composition and Inverse

16. Given $f(x)=\frac{5x+1}{2x-1}$ and $g(x)=\frac{3}{x-1}$, find $(g\circ f)(x)$. Then state the domain.

17. State whether the functions are one-to-one.

a. $f(x)=3x^2+x-7$ b. $g(x)=x^3-8$ c. $h(x)=|x-5|+6$

18. Determine if the functions are invertible and restrict the domain if necessary. Then determine the inverse and it's domain.

a. $f(x)=(x-3)^2+12$

Invertible? Yes No

Restricted domain (if necessary):

$f^{-1}(x)=$

Domain of $f^{-1}(x)$:

b. $f(x)=\frac{x+1}{2x-3}$

Invertible? Yes No

Restricted domain (if necessary):

$f^{-1}(x)=$

Domain of $f^{-1}(x)$:

c. $y=|x+3|-4$

Invertible? Yes No

Restricted domain (if necessary):

$f^{-1}(x)=$

Domain of $f^{-1}(x)$:

Exponential/Logarithmic Formulas Given: $A = P\left(1+\frac{r}{n}\right)^{nt}$ $A = Pe^{rt}$ $A(t) = A_0e^{kt}$

19. For the following functions determine the equation of the asymptote and end behavior using limits.

a. $f(x) = \left(\frac{1}{3}\right)^x - 7$ b. $f(x) = 3\ln(x+1)$ c. $f(x) = e^{x+4} - 3$

20. Condense the logarithmic expression. Simplify your result.

a. $2\log x - \log 3$ b. $5\ln x + \ln xy - 2\ln 3x$

21. Expand the logarithmic expression.

a. $\log_9 \frac{x^2}{13y^5}$ b. $\ln\left(\frac{3x^3}{2(x+1)^3(3x)}\right)$

22. Evaluate the expression.

a. $\log_{16}\frac{1}{4}$ b. $\ln e^4 - \ln\left(\frac{1}{e^2}\right)$ c. $\log_3 27$ d. $\log_{\frac{1}{2}} 32$

23. Simplify each expression

a. e^{8ln} b. $\ln e^{(2x+3)}$ c. $e^{5\ln x} + \ln e^{(12)}$

24. Solve. LEAVE EXACT, NON-CALCULATOR!

a. $8^{2x+3} = \left(\frac{1}{4}\right)^{x+1}$ b. $\log_9 4x = \log_9(2x+1) + \log_9(x)$ c. $\log_4(2x) + \log_4(x-2) = 2$

d. $2^x = 15$ e. $\ln(x+1) = 3$ f. $e^{2x+4} = 3$

25. A certain bacteria has a half-life of 39 days.
a) Determine the decay rate exactly and give both exact and decimal.

Rate (exact): ________________

b) Determine how many bacteria are left after 100 days if the initial sample had 700 bacteria.

Solution (exact):____________________________

26. A certain bacteria has a half-life of 55 days.
a) Determine the decay rate exactly and give both exact and decimal.

Rate (exact): ________________

b) Determine how many bacteria are left after 30 days if the initial sample had 806 bacteria.

Solution (exact):____________________________

<u>Trigonometry</u> Given Blank unit circle, sum and difference formulas, double angle formulas, Area formulas and Laws of Sines and Cosines.

27. If the point given is on the terminal side of θ, determine the exact value of all six trigonometric functions.

a. $(6, 8)$

b. $(2, -3)$

28. Name the quadrant in which the angle θ lies if:

a. $\cos\theta < 0,\ \csc\theta < 0$ ____________________

b. $\cot\theta < 0,\ \cos\theta > 0$ ____________________

c. $\sec\theta < 0,\ \tan\theta < 0$ ____________________

d. $\sin\theta > 0,\ \cos\theta > 0$ ____________________

29. What is the reference angle if $\theta = 247^0$

30. Give and expression for all angles that are coterminal with the given angle and then one positive and one negative coterminal angle.

a. $215°$

b. $\frac{7\pi}{15}$

c. $-\frac{4\pi}{7}$

All coterminal: ____________
Positive: ________________
Negative: _______________

All coterminal: ____________
Positive: ________________
Negative: _______________

All coterminal: ____________
Positive: ________________
Negative: _______________

31. Find the exact value of the 5 remaining trig functions if $sec\ \theta = \frac{9}{8}$ and $\csc\theta < 0$.

32. Find the exact value of the 5 remaining trig functions if $\sin\theta = -\frac{2}{3}$ and $\cot\theta > 0$.

33. If $\sin\theta = -\frac{4}{5}$ and $\pi \leq \theta < \frac{3\pi}{2}$ what is $\cos(2\theta)$?

34. If $\cos\theta = 5/13$ and $\frac{3\pi}{2} \leq \theta < 2\pi$ what is $\sin(2\theta)$?

35. Find the exact value of the expression

a. $\sec\left(\frac{4\pi}{3}\right)$

b. $\cot\left(-\frac{5\pi}{6}\right)$

c. $\tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$

d. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

e. $\sec^{-1}(-2)$

f. $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

g. $\tan\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

h. $\cos\left(\sin^{-1}\left(\frac{1}{4}\right)\right)$

i. $\sec^{-1}\left(\cot\left(\frac{3\pi}{4}\right)\right)$

j. $\cot(\pi) - \csc\left(\frac{\pi}{4}\right)$

k. $\cos^2\left(\frac{\pi}{6}\right)\sin\left(-\frac{5\pi}{3}\right)$

l. $\csc^2\left(\frac{5\pi}{6}\right) + \cot\left(\frac{5\pi}{4}\right)$

36. Given the functions below, find the amplitude, period, phase shift, and vertical shift.

a. $f(x) = 3\sin(3x - 8) + 4$ b. $f(x) = -\sin(x - 5)$ c. $f(x) = -\frac{2}{3}\sin\big(2(x + \pi)\big) - 3$

Amplitude: ______________ Amplitude: ______________ Amplitude: ______________

Period: ______________ Period: ______________ Period: ______________

Phase Shift: ______________ Phase Shift: ______________ Phase Shift: ______________

Vertical Shift: ______________ Vertical Shift: ______________ Vertical Shift: ______________

37. Find the exact value of cos 75º.

38. Find the exact value of tan 15º.

39. Find the exact value of $\cos(2\theta)$ and $\sin(2\theta)$, if $\cos\theta = \frac{2}{5}$ and $\csc\theta < 0$.

40. Find the exact value of $\cos(A + B)$ and $\sin(A - B)$, if $\tan A = \frac{3}{2}$ and A is in Q1. CosB=-2/5 and is in Q2.

41. Simplify the expressions. Reduce to a single trigonometric function that is not rational.

a. $1 - \frac{sin^2\theta}{1+\cos\theta}$ b. $\sin x\cos^2 x + \sin^3 x$ c. $\frac{1-\sec^2\theta}{\sin^2\theta}$

42. Verify the identity

a. $\cos(\frac{\pi}{2} + \theta) = -sin\,\theta$ b. $\sin^2 x\tan^2 x\csc^2 x + \cos^2 x\tan^2 x\csc^2 x = \sec^2 x$

c. $sec\ \theta = sin\ \theta(tan\ \theta + cot\ \theta)$

d. $\frac{\csc^2\theta - \cot^2\theta}{1-\sin^2\theta} = \sec^2\theta$

e. $\frac{1+\sin}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = \frac{2\cos(\theta)}{1-\sin\theta}$

f. $\frac{\sin\theta}{\csc\theta - 1} - \frac{\sin\theta}{\csc\theta+1} = 2\sin\theta\tan^2\theta$

43. Solve in the interval $[0, 2\pi)$

a. $6\ cos\ x - 3 = 0$

b. $(cot\ \theta + 1)\ (\ csc\ \theta - ½\ ) = 0$

c. $3\tan^2 x + 2 = 5$

d. $\cos x \sin x = 3\cos x$

e. $5 + 7\sin^2\theta = 8 + 3\sin^2\theta$

e. $\cos^2 x = 2\cos(x)$

<u>Vectors and Parametrics:</u> Formulas given: $x = tv_0\cos\theta$; $y = tv_0\sin\theta - \frac{1}{2}gt^2 + h_0$;

$$where\ g = 32\frac{ft}{s^2}\ or\ 9.8\frac{m}{s^2}$$

44. If **r** = $\langle 3, 9\rangle$ and **s** =$\langle -3,6\rangle$, determine the following:

a. 5**r** – 2**s**

b. $2r + \frac{1}{2}s$

c. $3(r - 2s)$

45. Let $\overrightarrow{AB}$ be the vector with given initial point A and terminal point B. Write $\overrightarrow{AB}$ as a linear combination of the vectors **i** and **j**.

a. $A(10, -4)$ and $B(-1, -3)$

b. $A(3, 0)$ and $B(2, -7)$

45. Find the magnitude of $\overrightarrow{AB}$ with initial point A and terminal point B.
a. $A(3, -7)$ and $B(8, -9)$ b. $A(-4, -2)$ and $B(4, 7)$

46. Find the component form of $\overrightarrow{AB}$ with initial point A and terminal point B.
a. $A(-12, 7)$ and $B(8, -2)$ b. $A(-2, -3)$ and $B(-1, -2)$.

47. Find the component form of v with the given magnitude and direction angle.

a. $|v| = 18, \theta = 240°$ Exact *b. $|v| = 5, \theta = 135°$

48. Find the direction angle of the vector to the nearest tenth. $p = -4i + 4j$

49. Write the following parametric equations in rectangular form:

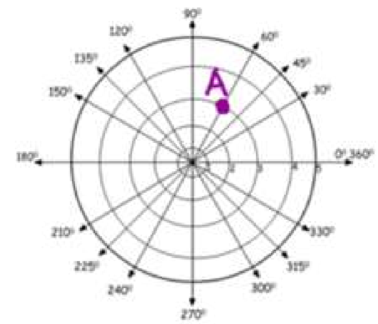
a. $x = 3t - 1;\ y = 2t^2 + 6$ b. $x = 4\cos\theta;\ y = 2\sin\theta$ c. $x(t) = 5t + 1;\ y(t) = 3t^2 - 4$

<u>Polar and Complex</u>: No additional formulas given! Should know/come up with:
$x = r\cos\theta, y = r\sin\theta,\ x^2 + y^2 = r^2$ *and* $\tan\theta = \frac{y}{x}$

50. Find the rectangular coordinates of:
a. $(4, 120°)$ b. $(-2, 3\pi/4)$

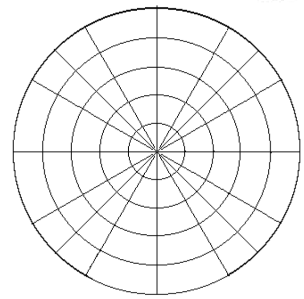
51. Find four sets of polar coordinates for the rectangular point $(-1, -1)$ if $-2\pi \le \theta \le 2\pi$.

52. Identify four polar coordinates for point A if $-2\pi \leq \theta \leq 2\pi$.



53. Plot the following polar points on the polar paper below.

$A\left(2, \frac{3\pi}{2}\right)$
$B\left(-3, \frac{5\pi}{6}\right)$
$C\left(0, \frac{\pi}{12}\right)$
$D\left(-3, -\frac{3\pi}{4}\right)$



54. Write the polar equations in rectangular form:

a. r = -6sinθ b. r = 2cosθ c. r = 5 d. $r \sin\theta = -7$

e. $r = -8$ f. $r = 6\cos\theta$ g. $\theta = \frac{2\pi}{3}$

55. Write the rectangular equations in polar form, then graph:

a. $x^2 + y^2 = 16$ b. $(x-2)^2 + y^2 = 4$ c. x = 4

Limits and Continuity

56. Determine whether each function is continuous at the given x-value(s). If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

a. $f(x) = \frac{x-2}{x+4}$; at $x = -4$ b. $f(x) = \frac{x+1}{x^2+3x+2}$; at $x = -1$ and $x = -2$

57. Evaluate each limit, if it exists.

a. $\lim\limits_{x \to 0^+} \left(4 - \sqrt{x}\right)$

b. $\lim\limits_{x \to 4} \dfrac{x^2 - 16}{x - 4}$

c. $\lim\limits_{x \to -1^+} \dfrac{1}{x+1}$

d. $\lim\limits_{x \to 3} \left(x^2 + 3x - 8\right)$

e. $\lim\limits_{x \to -6} \dfrac{x^2 - 36}{x + 6}$

f. $\lim\limits_{\theta \to \frac{\pi}{2}^-} \tan\theta$

g. $\lim\limits_{x \to -2} f(x)$ if $f(x) = \begin{cases} x - 1 & x < -2 \\ x^2 & x \geq -2 \end{cases}$

h. $\lim\limits_{x \to 5} f(x)$ if $f(x) = \begin{cases} x - 1 & x < 5 \\ 3 & x = 5 \\ x^2 - 4x - 1 & x > 5 \end{cases}$

i. $\lim\limits_{x \to \infty} x^4 - 3x^2 + 7$

j. $\lim\limits_{x \to -\infty} \frac{3x^2+7}{x^2-1}$

k. $\lim\limits_{x \to \infty} \ln(x)$

Calculator Practice:

Matrices and Complex

58. Evaluate each of the following using matrices A, B and C

$A = \begin{bmatrix} -1 & 5 \\ 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 2 & -1 \\ 0 & -5 & 3 \end{bmatrix}$ $C = \begin{bmatrix} -1 & 0 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

a. $AB + C$ b. $3AC - B$ c. $2(B+C)$

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Polynomials and Rationals:

59. Find ALL zeros. (real and/or complex) $f(x) = x^5 - 18x^3 + 30x^2 - 19x + 30$

Exponential/Logarithmic Formulas Given: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $A = Pe^{rt}$ $A(t) = A_0e^{kt}$

60. Suppose $1000 is invested in an account that compounds continuously at a rate of 4.25%. Determine how long it will take to double. Round to the nearest year.

61. A scientist has 37 grams of a radioactive substance that decays 30% continuously. How many grams of radioactive substance remain after 9 years? Round to nearest tenth.

62. A certain radioactive substance has a half-life of 2488 years. Find the decay rate in EXACT FORM. Then, if there was 100g initially, find the amount of substance left after 1000 years. Round to nearest tenth.

Trigonometry Given Blank unit circle, sum and difference formulas, double angle formulas, Area formulas and Laws of Sines and Cosines.

63. Two observes simultaneously measure the angle of elevation of a helicopter. One angle measured is A: 25^0 and the other is B: 40^0. If the observers are 100 feet apart and the helicopter lies over the line joining them. How far away from the helicopter are the observers A and B?

64. Solve the following triangles. Round to the nearest hundredth.

a. $a = 11cm,\ b = 6\ cm,\ A = 22°$

b. $a = 13\ m,\ b = 12\ m,\ c = 8m$

c. $a = 9\ cm,\ b = 10\ cm,\ C = 42°$

d. $a = 5\ cm,\ A = 36°,\ B = 42°$

e. $a = 25in,\ c = 18in,\ C = 63°,$

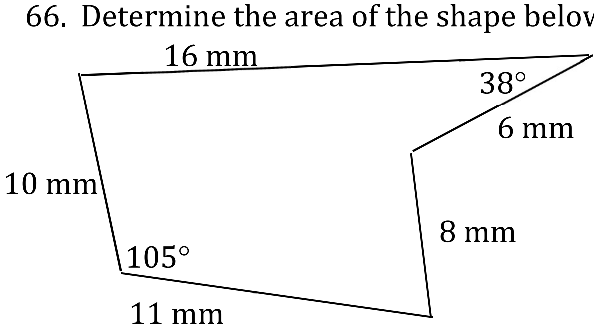
f. $B = 20°,\ a = 6mm,\ b = 4mm$

65. Determine the area of each triangle to the nearest tenth.

a. $A = 95°,\ b = 12m, c = 18\ m$

b. $a = 44, b = 47, c = 53$

66. Determine the area of the shape below with given dimensions.



<u>Vectors and Parametrics:</u> Formulas given: $x = tv_0 \cos\theta;\ y = tv_0 \sin\theta - \frac{1}{2}gt^2 + h_0;$

$$where\ g = 32\frac{ft}{s^2}\ or\ 9.8\frac{m}{s^2}$$

67. Find the direction angle. Round to the nearest degree, when necessary.
a. $\langle -2, 3\rangle$ b. $3\mathbf{i} - 3\mathbf{j}$

68. A plane takes off at 220 miles per hour at an angle of 51° with the ground. Find the magnitude of the horizontal and vertical components of its velocity. Round to the nearest tenth.

Horizontal Component: ___________ Vertical Component: ______________

69. Charles leaves his apartment and walks 55° north of west for 1000 feet and then walks 300 feet due north to go bowling. How far is Charles from his apartment?

70. Mrs. Parker can hit a tennis ball with an initial velocity of 65 feet per second at an angle of 20° to the horizontal at a height of 2.5 ft from the ground. She is 22 feet from a net of height 3. Round all answers to the *nearest hundredth.*

a) Write a set of parametric equations that describe the position of the ball as a function of time.

b) How high is the ball when it reaches the net?

c) Does the ball clear the net?

71. Suppose Mr. Shanazu hit a golf ball with an initial velocity of 150 feet per second at an angle of 30° to the horizontal. Round all answers to the *nearest hundredth.*

a) Write a set of parametric equations that describe the position of the ball as a function of time.

b) How long is the golf ball in the air?

c) When is the ball at its maximum height?

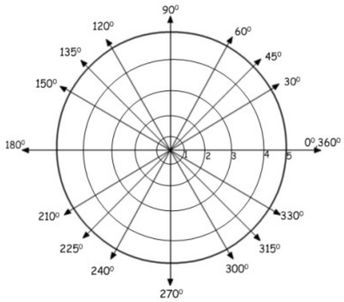
d) What is the maximum height of the golf ball?

e) His goal was to hit the golf ball at least 600 feet. Did he reach his goal? How far away did the golf ball land?

Polar

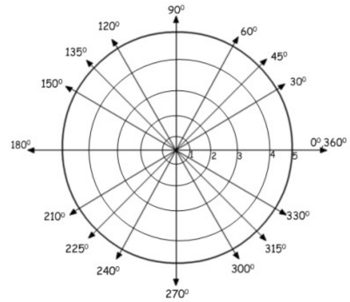
72. Graph the polar equations AND state the symmetry.

a. r = 3 + 3sinθ



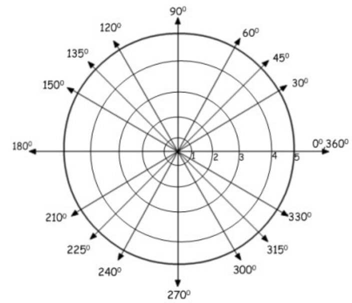
Symmetry: ________________

b. $\theta = -\pi/6$



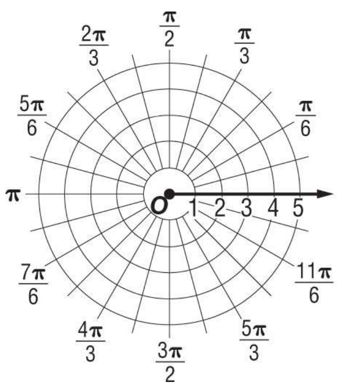
Symmetry: ________________

c. r = 5cos θ



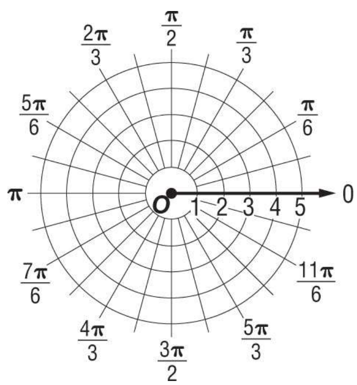
Symmetry: ________________

d. r = 2 – 2sinθ



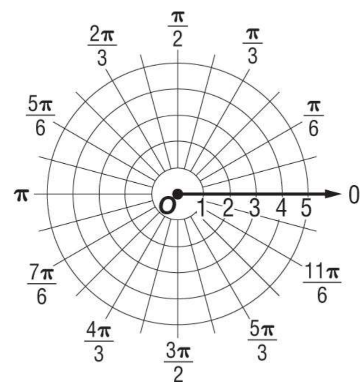
Symmetry: ________________

e. r = 3 + 2cosθ



Symmetry: ________________

f. r = 3 secθ



Symmetry: ________________