(dots or actual counters) and indicate that a set of 5 counters is one-fourth. How much is the set of 15 counters?

Top and Bottom Numbers

The way that we write fractions with a top and a bottom number and a bar between is a convention—an arbitrary agreement for how to represent fractions. (By the way, always write fractions with a horizontal bar, not a slanted one. Write $\frac{3}{4}$, not 3/4.) As a convention, it falls in the category of things that you simply tell students. However, a good idea is to make the convention so clear by way of demonstration that students will tell *you* what the top and bottom numbers stand for. The following procedure is recommended even if your students have been "using" symbolic fractions for several years.

Display several collections of fractional parts in a manner similar to those in Figure 5.10. Have students count the parts together. After each count, write the correct fraction, indicating that this is how it is written as a symbol. Include sets that are more than one but write them as simple or "improper" fractions and not as mixed numbers. Include at least two pairs of sets with the same top numbers such as $\frac{4}{8}$ and $\frac{4}{3}$. Likewise, include sets with the same bottom numbers. After the class has counted and you have written the fraction for at least six sets of fractional parts, pose the following questions:

- What does the bottom number in a fraction tell us?
- What does the top number in a fraction tell us?

STOP **Before reading further, answer these two questions in your own words. Don't rely on formulations you've heard before. Think in terms of what we have been talking about—namely, fractional parts and counting fractional parts. Imagine counting a set of 5 eighths and a set of 5 fourths and writing the fractions for these sets. Use children's language in your formulations and try to come up with a way to explain these meanings that has nothing to do with the type of model involved.**

Here are some reasonable explanations for the top and bottom numbers.

- *Top number:* This is the counting number. It tells how many shares or parts we have. It tells how many have been counted. It tells how many parts we are talking about. It counts the parts or shares.
- *Bottom number:* This tells what is being counted. It tells what fractional part is being counted. If it is a 4, it means we are counting *fourths;* if it is a 6, we are counting *sixths;* and so on.

This formulation of the meanings of the top and bottom numbers may seem unusual to you. It is often said that the top number tells "how many." (This phrase seems unfinished. How many *what?*) And the bottom tells "how many parts it takes to make a whole." This may be correct but can be misleading. For example, a $\frac{1}{6}$ piece is often cut from a cake without making any slices in the remaining $\frac{5}{6}$ of the cake. That the cake is only in two pieces does not change the fact that the piece taken is $\frac{1}{6}$. Or if a pizza is cut in 12 pieces, two pieces still make $\frac{1}{6}$ of the pizza. In neither of these instances does the bottom number tell how many pieces make a whole.

There is evidence that an iterative notion of fractions, one that views a fraction such as $\frac{3}{4}$ as a count of three things called *fourths,* is an important idea for children to develop. The iterative concept is most clear when focusing on these two ideas about fraction symbols:

- The top number *counts.*
- The bottom number tells *what is being counted.*

The *what* of fractions are the fractional parts. They can be counted. Fraction symbols are just a shorthand for saying *how many* and *what.*

Smith (2002) points out a slightly more "mathematical" definition of the top and bottom numbers that is completely in accord with the one we've just discussed. For Smith, it is important to see the bottom number as the divisor and the top as the multiplier. That is, $\frac{3}{4}$ is three *times* what you get when you *divide* a whole into four parts. This multiplier and divisor idea is especially useful when students are asked later to think of fractions as an indicated division; that is, $\frac{3}{4}$ also means $3 \div 4$.

Numerator and Denominator

To count a set is to *enumerate* it. The common name for the top number in a fraction is the *numerator.*

A denomination is the name of a class or type of thing. A \$1 bill, a \$5 bill, and a \$10 bill are said to be bills of different *denominations*. The common name for the bottom number in a fraction is the *denominator.*

The words *numerator* and *denominator* have no common reference for children. Whether these words are used or not, the words themselves will not help young students understand the meanings.

Mixed Numbers and Improper Fractions

If you have counted fractional parts beyond a whole, your students already know how to write $\frac{13}{16}$ or $\frac{13}{3}$. Ask, "What is another way that you could say 13 *sixths?*" Students may suggest "two wholes and one-sixth more," or "two plus one-sixth." Explain that these are correct and that $2 + \frac{1}{6}$ is usually written as $2\frac{1}{6}$ and is called a *mixed number.* Note that this is a symbolism convention and must be explained to students. What is not at all necessary is to teach a rule for converting mixed numbers to common fractions and the reverse. Rather, consider the following task.

ACTIVITY 5.4

Mixed-Number Names

Give students a mixed number such as $3\frac{2}{5}$. Their task is to find a single fraction that names the same amount. They may use any familiar materials or make drawings, but they must be able to give an explanation for their result. Similarly, have students start with a fraction greater than 1, such as $\frac{17}{4}$, and have them determine the mixed number and provide a justification for their result.

Repeat the "Mixed-Number Names" task several times with different fractions. After a while, challenge students to figure out the new fraction name without the use