whole—the parts that result when the whole or unit has been partitioned into *equal-sized portions* or *fair shares*.

Children seem to understand the idea of separating a quantity into two or more parts to be shared fairly among friends. They eventually make connections between the idea of fair shares and fractional parts. Sharing tasks are, therefore, good places to begin the development of fractions.

Sharing Tasks

Considerable research has been done with children from first through eighth grades to determine how they go about the process of forming fair shares and how the tasks posed to students influence their responses (e.g., Empson, 2002; Lamon, 1996; Mack, 2001; Pothier & Sawada, 1983).

Sharing tasks are generally posed in the form of a simple story problem. *Suppose there are four square brownies to be shared among three children so that each child gets the same amount. How much (or show how much) will each child get?* Task difficulty changes with the numbers involved, the types of things to be shared (regions such as brownies, discrete objects such as pieces of chewing gum), and the presence or use of a model.

Students initially perform sharing tasks (division) by distributing items one at a time. When this process leaves leftover pieces, it is much easier to think of sharing them fairly if the items can be subdivided. Typical "regions" to share are brownies (rectangles), sandwiches, pizzas, crackers, cake, candy bars, and so on. The problems and variations that follow are adapted from Empson (2002).

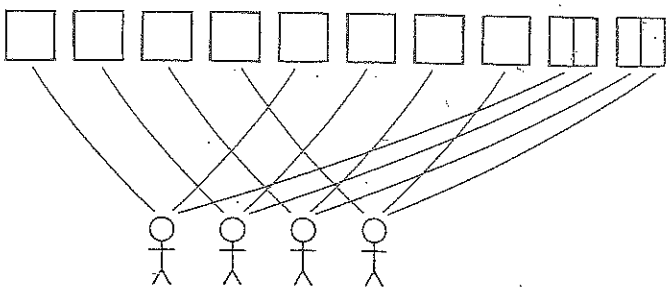
> Four children are sharing 10 brownies so that each one will get the same amount. How much can each child have?

Problem difficulty is determined by the relationship between the number of things to be shared and the number of sharers. Because students' initial strategies for sharing involve halving, a good place to begin is with two, four, or even eight sharers. For ten brownies and four sharers, many children will deal out two to each child and then halve each of the remaining brownies. (See Figure 5.1.)

Consider these variations in numbers:

5 brownies shared with 2 children
4 brownies shared with 8 children
2 brownies shared with 4 children
3 brownies shared with 4 children
5 brownies shared with 4 children

FIGURE 5.1
Ten brownies shared with four students.



Try drawing pictures for each of the preceding sharing tasks. Which do you think is most difficult? Which of these represents essentially the same degree of difficulty? What other tasks involving two, four, or eight sharers would you consider as similar, easier, or more difficult than these?

When the numbers allow for some items to be distributed whole (five shared with two), some students will first share whole items and then cut up the leftovers. Others will slice every piece in half and then distribute the halves. When there are more sharers than items, some partitioning must happen at the beginning of the solution process.

When students who are still using a halving strategy try to share five things among four children, they will eventually get down to two halves to give to four children. For some, the solution is to cut each half in half; that is, "each child gets a whole (or two halves) and a half of a half."

It is a progression to move to three or six sharers because this will force students to confront their halving strategies.

Try solving the following variations using drawings. Can you do them in different ways?

4 pizzas shared with 6 children

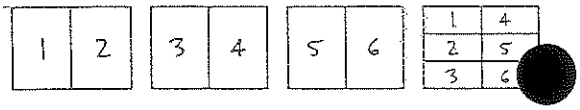
7 pizzas shared with 6 children

5 pizzas shared with 3 children

To subdivide a region into a number of parts other than a power of two (four, eight, etc.) requires an odd subdivision at some point. Several types of sharing solutions might be observed. Figure 5.2 shows some different approaches.

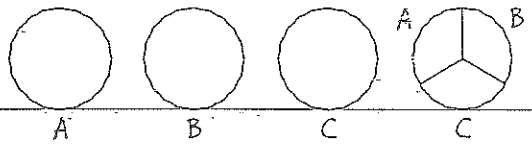
Use a variety of representations for these problems. The items to be shared can be drawn on worksheets as rectangles or circles along with a statement of the problem. Another possibility is to cut out construction paper circles or squares. Some students may need to cut and physically distribute the pieces. Students can use connecting cubes to make bars that they can separate into pieces. Or they can use more traditional fraction models such as circular "pie" pieces.

(a) Four candy bars shared with six children:



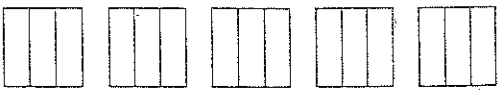
Cut all the bars in half.
Cut the last two halves into three parts.
Each child gets a half and sixth.

(b) Four pizzas shared with three children:



Pass out whole pizzas.
Cut the last pizza in three parts.
Each child gets 1 whole and one-third.

(c) Five sandwiches shared with three children:



Cut each sandwich in three parts (thirds).
Each child gets five parts—five-thirds.

FIGURE 5.2

Three different sharing processes.

Models for Fractions

There is substantial evidence to suggest that the use of models in fraction tasks is important. Unfortunately, many teachers in the upper grades, where manipulative materials are not as common, fail to use models for fraction development. Models can help