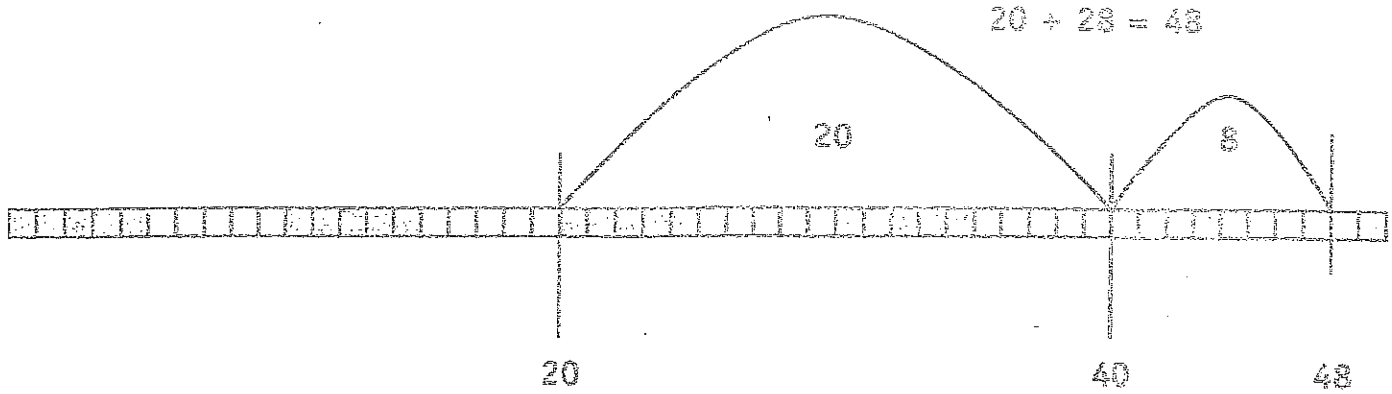
Gr. 3 U3 L11



The Open Number Line with Connecting Cubes

In contrast to a number line with counting numbers written below, an open number line is just an empty line used to record children's addition and subtraction strategies. Only the numbers children use are recorded, and the operations are recorded as leaps, or jumps. Representations on a number line can help children move beyond tedious strategies such as counting by ones to more efficient strategies such as taking leaps of ten, decomposing numbers, and using landmark numbers. Initially, a train of cubes of two colors in alternating groups of five cubes of each color is used as a bridge to support children who may still need to count by ones. The train of cubes allows for counting by ones if needed. By using groups of five in alternating colors, the five-structure is also provided as a support and thus it provides a bridge to the open number line where only the numbers used in children's strategies are recorded. This section (B1 – B15) makes use of the connecting cubes. The following section (C1 – C32) uses only the open number line.

Materials Needed for this Section

Train of 100 connecting cubes, using two colors arranged in alternating groups of five cubes of each color, stretched across the chalkboard (small magnets attached with Velcro® allow the train to adhere to a magnetic board or you can push thin wire through the train and tie it at each end of the board).

String · B1

Addition, Keeping One Number Whole and Taking Leaps of Ten

This string of related problems encourages children to keep one number whole and take leaps of ten. Use a train of 100 connecting cubes of two colors in alternating groups of five cubes of each color, stretched across the chalkboard (or whiteboard). Do one problem at a time. Record children's strategies by drawing the leaps as lengths on the chalkboard and use the cubes to check if needed, inviting children to comment on the representations. See Inside One Classroom, page ~~19~~ 20. If you notice children beginning to make use of the tens, invite a discussion on why this strategy is helpful. If the class comes to a consensus that it is a useful strategy when adding, you might want to make a sign about this and post it above the meeting area.

26 + 10

26 + 12

26 + 22

44 + 30

44 + 39

58 + 21

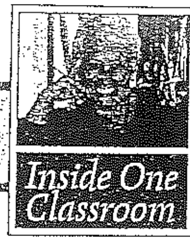
63 + 29

Behind the Numbers: How the String was Crafted

The first problem is the scaffold for the second. Assuming that children know the pattern of adding 10, this string encourages them to use it by starting the string with it. The second expression has a value just 2 greater than the first, and the leap can be added to the number line representation of the first problem. The third expression has a value 10 greater than the second and once again the initial representation can be used — but only if children offer this strategy. The next two problems are paired, since 39 is 9 more than 30. This problem also opens the door to thinking of 39 as 40 − 1. The last two problems support children in exploring both strategies: using the tens first and then adding the units, and taking a larger leap and subtracting at the end.

Gr 3 U2 L21 T

A Portion of the Minilesson

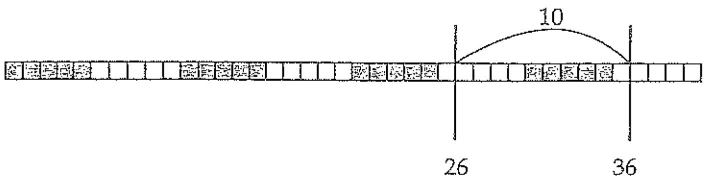


Author's Notes

Julie (the teacher): Here's our first warm-up problem: 26 + 10. Thumbs-up when you have an answer. Michael?

Michael: It's 36. I just know.

Julie: *(Draws the following representation of Michael's strategy:)*

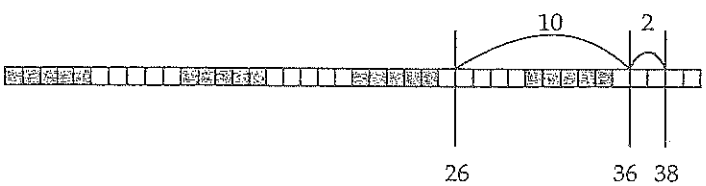


Julie places the train of cubes with magnets against the chalkboard and records Michael's strategy as a leap of ten.

Does everyone agree with Michael? *(No disagreement is apparent.)* OK. Let's go to the next one: 26 + 12. Emmy?

Emmy: I added a jump of 2 more onto the last problem.

Julie: Let's see what that looks like on the number line. *(Records the jumps in a different color on Michael's number line.)*



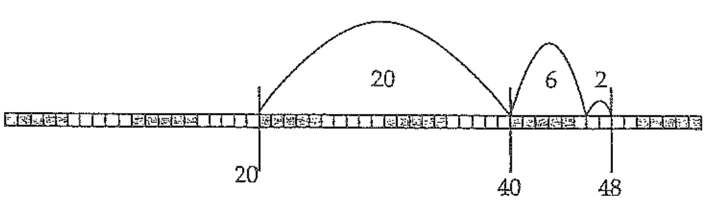
Since Emmy added two, Julie uses the same representation and just adds a jump of two.

Julie encourages the children to examine the efficiency of the strategy.

What do you think? Does Emmy's way work? Saves a lot of counting, doesn't it, if we think of 12 as 10 and 2? OK, let's go on to the next problem, 26 + 22. Show me with thumbs-up when you're ready. Susie.

Susie: It's 48, I think. I did 20 + 20 + 8.

Julie: Nice. You used the friendly number of 20 and added them. Let me record your strategy.



Did anybody do it a different way? Did anyone do what Emmy did last time and use the problem before?

Michael: I did. I just added on 10 more. So 38, then 48.

continued on next page

continued from previous page

Julie: Nice. It is really helpful to keep one number whole, isn't it? And just add on. Here's the next one: 44 + 30. Remember how we did all the patterns when we counted around the circle? See if that idea would be helpful here.

Alternative strategies are solicited but emphasis is placed on keeping one number whole and adding on in leaps of ten.

Abbie: It is! It's just 54, 64, 74.

Julie: How many of you agree with Abbie? Let's think about this as we continue with our string. If we agree at the end, we can make a sign and post it on our "Helpful Strategies" wall.

String · B2

Addition, Keeping One Number Whole and Taking Leaps of Ten

Like B1, this string encourages children to keep one number whole and take leaps of tens. Do one problem at a time; and record children's strategies by drawing the leaps on the chalkboard and use the cubes to check if needed, inviting other children to comment on the representations. See page 19 for further details.

43 + 20

43 + 24

43 + 44

52 + 30

52 + 39

68 + 22

68 + 29

Behind the Numbers: How the String was Crafted

The first problem is the scaffold for the second. Assuming children know the pattern of adding 10, the string encourages them to use it by starting the string with adding 2 tens. The value of the expression in the second problem is just 4 greater than the value of the first, and the leap can be added to the number line representation of the first problem. The third expression has a value 20 greater than the second and once again the initial representation can be used. The next two problems are paired, since 39 is just 9 more than 30. This problem also opens the door to thinking of 39 as 40 − 1. The last two problems support children in exploring both strategies: using the tens first and then adding the units, and taking a larger leap and subtracting at the end. For additional support, see the related strings that follow—B3 through B5.

Subtracting by Counting Up

This is an amazingly powerful way to subtract. Students working on the *think-addition* strategy for their basic facts can also be solving problems with larger numbers. The concept is the same. It is important to use *join with change unknown* problems or *missing-part* problems to encourage the counting-up strategy. Here is an example of each.

> Sam had 46 baseball cards. He went to a card show and got some more cards for his collection. Now he has 73 cards. How many cards did Sam buy at the card show?

> Juanita counted all of her crayons. Some were broken and some not. She had 73 crayons in all. 46 crayons were not broken. How many were broken?

The numbers in these problems are used in the strategies illustrated in Figure 4.5.

Emphasize the value of using tens by posing problems involving multiples of 10. In 50 – 17, the use of ten can happen by adding up from 17 to 20, or by adding 30 to 17. Some students may reason that it must be 30-something because 30 and 17 is less than 50, and 40 and 17 is more than 50. Because it takes 3 to go with 7 to make 10, the answer must be 33. Work on naming the missing part of 50 or 100 is also valuable. (See Activity 2.18, "The Other Part of 100," p. 54.)

Take-Away Subtraction

Take-away methods are more difficult to do mentally or even with the help of paper and pencil. This is especially true when problems involve three digits. Exceptions involve problems such as 423 – 8 or 576 – 300 (subtracting a number less than 10 or a multiple of 10 or 100). However, take-away strategies are bound to occur, probably because traditional textbooks emphasize take-away as the meaning of subtraction. Take-

FIGURE 4.5

Subtraction by counting up is a powerful method.

Invented Strategies for Subtraction by Counting Up

Add Tens to Get Close, Then Ones

73 – 46

46 and 20 is 66.
(30 more is too much.)
Then 4 more is 70 and 3 is 73.
That's 20 and 7 or 27.

$46 > 20$
$66 > 4$
$70 > 3$
73 $\quad 27$

Add Tens to Overshoot, Then Come Back

73 – 46

46 and 30 is 76.
That's 3 too much, so it's 27.

73 – 46
$46 + 30 \to 76 - 3 \to 73$
$30 - 3 = 27$

Add Ones to Make a Ten, Then Tens and Ones

73 – 46

46 and 4 is 50.
50 and 20 is 70 and 3 more is 73. The 4 and 3 is 7 and 20 is 27.

73 – 46
$46 + 4 \to 50$
$+ 20 \to 70$
$+ 3 \to 73$
27

Similarly,

46 and 4 is 50.
50 and 23 is 73.
23 and 4 is 27.

$46 + 4 \to 50$
$50 + 23 \to 73$
$23 + 4 = 27$

away is very likely the strategy that will come to mind first for students who have previously been taught the traditional algorithm.

Four take-away strategies are shown in Figure 4.6, and these should not be discouraged. We suggest, however, that you emphasize adding-on methods whenever possible.

There were 73 children on the playground. The 46 third-grade students came in first. How many children were still outside?

The two methods that begin by taking tens from tens are reflective of what most students do with base-ten pieces. The other two methods leave one of the numbers intact and subtract from it. Try 83 – 29 in your head by first taking away 30 and adding 1 back. This is a good mental method when subtracting a number that is close to a multiple of ten.

Try computing 82 – 57. Use both take-away and counting up methods. Can you use all of the strategies in Figures 4.5 and 4.6 without looking?

Extensions and Challenges

Each of the examples in the preceding sections involved sums less than 100 and all involved *bridging a ten;* that is, if done with a traditional algorithm, they require carrying or borrowing. Bridging, the size of the numbers, and the potential for doing problems mentally are all issues to consider.

Invented Strategies for Take-Away Subtraction

Take Tens from the Tens, Then Subtract Ones	Take Away Tens, Then Ones
73 – 46 70 minus 40 is 30. Take away 6 more is 24. Now add in the 3 ones — 27. 73 – 46 70 – 40 → 30 – 6 → 24 + 3 → 27	73 – 46 73 minus 40 is 33. Then take away 6: 3 makes 30 and 3 more is 27. 73 – 40 → 33 – 3 30 – 3 → 27
Or	**Take Extra Tens, Then Add Back** 73 – 46 73 take away 50 is 23. That's 4 too many. 23 and 4 is 27. 73 – 50 → 23 + 4 27
70 minus 40 is 30. I can take those 3 away, but I need 3 more from the 30 to make 27. $\begin{array}{r} \cancel{73} \\ -46 \\ \hline 30 \\ -3 \\ \hline 27 \end{array}$	**Add to the Whole If Necessary** 73 – 46 Give 3 to 73 to make 76. 76 take away 46 is 30. Now give 3 back — 27. +3 73 – 46 76 – 46 → 30 – 3 → 27

FIGURE 4.6

Take-away strategies work reasonably well for two-digit problems. They are a bit more difficult with three digits.