

Pictures

Carefully designed pictures can support the development of important strategies for multiplication and division by building in potentially realizable strategies or constraints. (Many of the pictures in this first section of the guide are used with permission of the Freudenthal Institute.)

Materials Needed for this Section

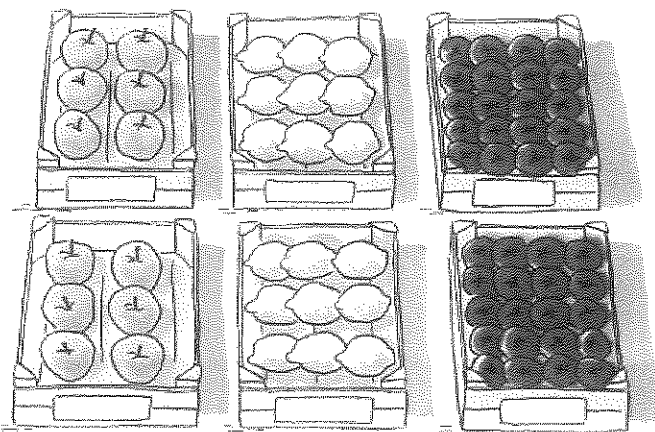
Overhead projector
and overhead marker

Overhead
transparencies of
Appendixes A—J

Boxes of Fruit · A1

Counting by Ones, Skip-Counting, Repeated Addition, Arrays

How many apples? How many lemons? How many tomatoes? Show one type of fruit at a time, using an overhead transparency of Appendix A, covering the other boxes. Invite discussion and introduce the multiplication notation as students share their strategies. For example, if a student says, "I see 2 sixes, so I know 6 plus 6 is 12," you might circle the groups of six and write: $2 \times 6 = 6 + 6 = 12$. If a student skip-counts by threes, mark the groups and write 3, 6, 9, 12. Then write $4 \times 3 = 12$.



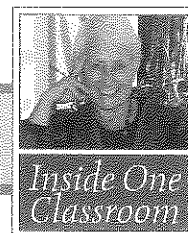
Behind the Numbers: How the String was Crafted

The fruit has been arranged in arrays and in groups to subtly suggest skip-counting or repeated addition. Some students may skip count the apples by twos; others by threes, and still others by sixes. The lemons may be mathematized in groups of three (horizontally) or six (vertically), or as $9 + 9$. The boxes of tomatoes make use of five-times and ten-times. Two groups of five may be put together to make a group of ten. Repeated addition of tens now may be used. If students need to count by ones, however, they can, since every piece of fruit is shown.

A Portion of the Minilesson

Willem (the teacher): I passed a fruit stand the other day as I was walking and this is what I saw: apples, lemons, and tomatoes. Let's just look at the apples first (*covers the other two boxes*). How many apples are there? Give a "thumbs-up" signal when you know (*provides think time until most thumbs are up*). Helena?

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Author's Notes

Willem provides a context as he introduces the fruit boxes. Then he focuses on the apples. Think time is provided to allow everyone to be engaged.

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Helena: Twelve. I saw 6 plus 6. Six in each box.

Willem: *(Writes $2 \times 6 = 6 + 6$ and circles the 2 groups of 6 that Helena describes).* So you saw 2 boxes with 6 in each and you thought about that as $6 + 6$. Did anybody do it a different way? Hans?

Willem encourages a variety of strategies to be shared.

Hans: I saw 6 and 6, too, but my sixes were the long way.

Willem: That's interesting. We can see the sixes this way, too.
(Circles 2 columns of 6). Did anyone do it a different way? Petra?

The groups under discussion are circled and recorded to support a variety of ways to mathematize the situation. In this way students are supported to group flexibly and relationships such as $4 \times 3 = 2 \times 6$ are being developed. Over time these relationships will become helpful as students automatize the multiplication facts.

Petra: I saw 4 threes. And I skip-counted, 3, 6, 9, 12.

Willem: *(Draws a circle around each of the groups of 3 and writes 3, 6, 9, 12).*
So you saw 4 groups of 3. I'm going to write that as 4×3 , ok? Danielle?

Danielle: I skip-counted, too. I did 2, 4, 6, 8, 10, 12.

Willem: Wow. So there are lots of nice ways to figure out the apples.
Let's look at the lemons now and see if we can find some nice ways for those.

Bags of Apples · A2

Skip-Counting, Distributive Property, Using Partial Products

Apples cost \$5 a bag. What is the cost for 3 bags? What is the cost for 5 bags? What is the cost for 8 bags? Use an overhead transparency of Appendix B. Show the groupings one at a time and invite discussion of strategies, representing what students say with multiplication notation.

