Fairfield Public Schools

Math Modeling 42

MATH MODELING 42A
The fundamental purpose of this Math Modeling 42 course is to formalize and extend the mathematics that students learned in Algebra I and Algebra II. This course presents an introduction to mathematical modeling based upon the use of elementary functions to describe and explore real-world data and phenomena. In this course, students will demonstrate graphical, numerical, symbolic, and verbal approaches to the investigation of data, functions, equations, and models. The emphasis will be on interesting applications of elementary mathematics together with the ability to construct useful mathematical models and analyze them critically and to communicate quantitative concepts effectively.

Pacing Guide							
1st Marking Period				2nd Marking Period			
Unit 1 **Functions and Mathematical Models** 2 weeks	Unit 2 **Linear Functions and Models** 2 weeks	Unit 3 **Natural Growth Models** 2 weeks	Unit 4 **Continuous Growth and Logarithmic Models** 2 weeks	Unit 5 **Quadratic Functions and Models** 2 weeks	Unit 6 **Polynomial Models and Linear Systems** 2 weeks	Unit 7 **Bounded Growth Models** 2 weeks	FINAL

Course Overview		
Central Understandings Insights learned from exploring generalizations through the essential questions. (Students will understand that…) • Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technologies. • Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. • Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools, and technologies. • Data can be analyzed to make informed decisions using a variety of strategies, tools, and technologies.	**Essential Questions** • How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? • How are quantitative relationships represented by numbers? • How do geometric relationships and measurements help us to solve problems and make sense of our world? • How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decisions?	**Assessments** • Formative Assessments • Summative Assessments

Content Outline	Standards
I. Unit 1 – Functions and Mathematical Models II. Unit 2 – Linear Functions and Models III. Unit 3 – Natural Growth Models IV. Unit 4 – Continuous Growth and Logarithmic Models V. Unit 5 – Quadratic Functions and Models VI. Unit 6 - Polynomial Models and Linear Systems VII. Unit 7 – Bounded Growth Models VIII. Unit 8 – Trigonometric Models	Connecticut Common Core State Standards are met in the following areas: • ***The Number Systems*** • ***Expressions and Equations*** • ***Number and Quantity*** • ***Algebra*** • ***Functions*** • ***Statistics and Probability***

Math Modeling 42 Standards for Mathematical Practice The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:	
Standards	***Explanations and Examples***
1. Make sense of problems and persevere in solving them.	In Algebra, students solve problems involving equations and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2. Reason abstractly and quantitatively.	This practice standard refers to one of the hallmarks of algebraic reasoning, the process of de-contextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems and then transforming the models via algebraic calculations (A-SSE, A-APR, F-IF) to reveal properties of the problems.
3. Construct viable arguments and critique the reasoning of others.	In Algebra, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Indeed, other mathematical practices in Algebra I might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.
5. Use appropriate tools strategically.	Students consider available tools such as spreadsheets, a function modeling language, graphing tools and many other technologies so they can strategically gain understanding of the ideas expressed by individual content standards and to model with mathematics.
6. Attend to precision.	In Algebra, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely helps students understand the idea in new ways.
7. Look for and make use of structure.	In Algebra, students should look for various structural patterns that can help them understand a problem. For example, writing $49x^2 + 35x + 6$ as $(7x)^2 + 5(7x) + 6$ is a practice many teachers refer to as "chunking,'' highlights the structural similarity between this expression and $z^2 + 5z + 6$, leading to a factorization of the original: $((7x) + 3)\ ((7x) + 2)$.
8. Look for and express regularity in repeated reasoning.	Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism. For example, when comparing two different text messaging plans, many students who can compute the cost for a given number of minutes have a hard time writing general formulas that express the cost of each plan for *any* number of minutes. Constructing these formulas can be facilitated by methodically calculating the cost for several different input values and then expressing the steps in the calculation, first in words and then in algebraic symbols. Once such expressions are obtained, students can find the break-even point for the two plans, graph the total cost against the number of messages sent and make a complete analysis of the two plans.

Unit 1 – Modeling with Functions, 2 weeks top

In this unit, students will be introduced to the mathematical concept of a function. The focus will be on functions defined by tables, graphs, and rules. Additionally, students will explore increasing and decreasing functions.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table. • Functions are a single-valued mapping from one set-the domain of the function-to another-its range.	• What is a function? • What are different ways functions can be represented? • What information can you get about how a function changes in different formats?

Common Core State Standards

FUNCTIONS

Interpreting Functions

Understand the concept of a function and use function notation.

F-IF.1

Understand that a function is from one set (called the domain) to another set (called the range) assigns each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F-IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

F-IF.4

For a function that models a relationship between two quantities, interpret key features of graph and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.6

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Unit 2 – Linear Functions, 2 weeks [top]	
In this unit, the students will investigate situations that involve data modeled around the linear function. Students will use their graphing calculators to determine the linear regression function from a set of data, then use this information to make predictions.	
Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table. • Linear functions are characterized by a constant average rate of change (or constant additive change). • Reasoning about the similarity of "slope triangles" allows deducing that linear functions have a constant rate of change and a formula of the type $f(x) = mx + b$	• How can linear functions be modeled by data? • How can a function involving the combination of two linear be used to solve problems?

Common Core State Standards

FUNCTIONS

Interpreting Functions

Analyze functions using different representations

F-IF.7a

Graph linear and piecewise functions and show intercepts. Graph by hand for simple cases and using technology for more complicated cases

Building Functions

Build a function that models a relationship between two quantities

F-BF.1.Fairfield

Write a linear function that describes a relationship between two quantities through the use of technology (i.e., linear regression).

Linear, Quadratic, and Exponential Models

Construct linear models and solve problems

F-LE.1a

Prove that linear functions grow by equal differences over equal intervals.

F-LE.1b

Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F-LE.2

Construct linear functions given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expressions for functions in terms of the situation they model

F-LE.5

Interpret the parameters in linear functions in terms of the context.

Unit 3 – Natural Growth Models, 2 weeks top

In this unit, the students will study functions that model exponential growth models. From this information, the students will again use their calculators to create a natural growth model, which then allows the students to make predictions. The students will then illustrate their applicability to a wide variety of real-world situations.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table. • Exponential functions are characterized by a rate of change that is proportional to the value of the function. It is property of exponential functions that whenever the input is increased by 1 unit, the output is multiplied by a constant factor.	• How is a natural growth model different than a linear model? • How can a natural growth model be used to make predictions and solve problems? • What is the difference between exponential growth and exponential decay? How is this exemplified in a mathematical model?

Common Core State Standards

FUNCTIONS
Interpreting Functions
Analyze functions using different representations

F-IF.7e
Graph exponential functions (only for percent growth or decay), showing intercepts and end behavior. Graph by hand for simple cases and using technology for more complicated cases.

F-IF.8b
Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $(0.97)^t$, $(1.01)^{12t}$, $(1.2)^{t/10}$ *and classify them as representing exponential growth or decay.*

Building Functions
Build a function that models a relationship between two quantities

F-BF.1.Fairfield
Write an exponential growth and decay functions that describe a relationship between two quantities (only for percent growth or decay) through the use of technology (i.e., exponential regression).

Linear, Quadratic, and Exponential Models
Construct linear models and solve problems

F-LE.1c
Recognize situations in which one quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.2
Construct exponential functions given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expressions for functions in terms of the situation they model

F-LE.5.Fairfield

Interpret the parameters in exponential functions in terms of the context.

Unit 4 – Continuous Growth and Logarithmic Models, 2 weeks top	
In this unit, the focus will shift from compound growth models in unit 3 to continuous growth models. The students will learn that the continuous growth model shifts to the development of the number *e*. From this development, the students then are able to then create growth or decay models at any given moment. After the students work and solve continuous exponential growth and decay problems, the shift will be on the inverse of these functions, i.e., logarithmic functions. The development of the logarithmic function allows the students to solve problems in which the unknown is in the power.	
Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table. • Logarithmic functions are inverse functions of exponential functions and provide models f .	• What is the difference between compound and continuous growth/decay models? • How are logarithms and exponentials similar and different?

Common Core State Standards

FUNCTIONS

Interpreting Functions

Analyze functions using different representations

F-IF.7e

Graph exponential (only for continuous growth or decay) and logarithmic functions, showing intercepts and end behavior. Graph by hand for simple cases and using technology for more complicated cases

Building Functions

Build a function that models a relationship between two quantities

F-BF.1.Fairfield

Write an exponential growth and decay functions that describe a relationship between two quantities (only for continuous growth or decay) through the use of technology (i.e., exponential regression).

Linear, Quadratic, and Exponential Models

Construct linear models and solve problems

F-LE.4

For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

F-LE.5.Fairfield

Interpret the parameters in exponential functions in terms of the context.

Unit 5 – Quadratic Functions and Models, 2 weeks top

After unit 4, the emphasis of the course shifts to quadratic functions. In this unit, students will model various situations that involve a parabolic function. For example, the height of a ball at any given time when it is thrown. Students again will use their calculators to create quadratic models to make predictions and solve problems.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table. • Quadratic functions are characterized by a linear rate of change, so the rate of change of the rate of change is constant. • Reasoning about the vertex from of a quadratic function allows deducing that the quadratic has a minimum or maximum value and that if the zeros are real, they are symmetric about the *x*-coordinate of the maximum or minimum point.	• How are quadratic functions different than exponential and linear functions? • In what ways can quadratic functions be used to solve problems? • How do you find the maximum/minimum of quadratic functions? • How do you find the intercepts (x & y) for a quadratic function?

Common Core State Standards

FUNCTIONS

Interpreting Functions

Analyze functions using different representations

F-IF.7a.Fairfield
Graph quadratic functions and show intercepts, maxima, minima, and symmetry. Graph by hand for simple cases and using technology for more complicated cases.

F-IF.8a.Fairfield
Use the quadratic formula in a quadratic function to show zeros. Interpret these in context.

F-IF.8c.Fairfield
Determine the symmetry of a quadratic function.

Building Functions

Build a function that models a relationship between two quantities

F-BF.1.Fairfield
Write a quadratic function that describes a relationship between two quantities through the use of technology (i.e., quadratic regression).

Interpret expressions for functions in terms of the situation they model

F-LE.5.Fairfield
Interpret the parameters in quadratic functions in terms of the context.

Unit 6 – Polynomial Models and Linear Systems, 2 weeks top

In this unit, the students will build on their understanding of quadratic functions to polynomial functions. This development extends beyond the second-degree functions to include third-, fourth-, and beyond functions. The last part of this unit switches to the students solving system of equations. The students will learn that there are many different approaches to solving systems of linear equations, each determined by the format the equations are in.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table.	• How are different polynomial functions similar and different than the family of quadratic functions? • What does the number of solutions (none, one or infinite) of a system of linear equations represent? • What are the advantages and disadvantages of solving a system of linear equations graphically versus algebraically?

Common Core State Standards

FUNCTIONS
Interpreting Functions
Analyze functions using different representations

F-IF.7c
Graph polynomial functions through the use of technology, identifying zeros when suitable factorizations are available, and showing end behavior.

Interpret expressions for functions in terms of the situation they model

F-LE.5.Fairfield
Interpret the parameters in polynomial functions in terms of the context.

Building Functions
Build a function that models a relationship between two quantities

F-BF.1.Fairfield
Write a polynomial functions that describe a relationship between two quantities through the use of technology (i.e., cubic or quartic regression).

ALGEBRA
Reasoning with Equations and Inequalities
Solve systems of equations

A-REI.6
Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pares of linear equations in two variables.

A-REI.8
Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI.9
Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices).

Unit 7 – Bounded Growth Models, 2 weeks top	
In this last unit, the students will be studying the various functions that are bounded by a specific factor. For example, the logistic function is used to model situation in which a growth model does not extend up or down infinitely. Again, like most other units, technology will be used to create a mathematical model, make predictions, and solve problems.	
Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table.	• What are similarities and differences between continuous growth models and bounded growth models?

Common Core State Standards

FUNCTIONS

Interpreting Functions

Analyze functions using different representations

F-IF.7f.Fairfield

Graph logistic functions through the use of technology, identifying parameters such as carrying capacity, initial amount, and rate.

Interpret expressions for functions in terms of the situation they model

F-LE.5.Fairfield

Interpret the parameters in logistic functions in terms of the context.

Building Functions

Build a function that models a relationship between two quantities

F-BF.1.Fairfield

Write a logistic function that describes a relationship between two quantities through the use of technology (i.e., logistic regression).