Algebra 2 Summer Work Packet
Review and Study Guide

This study guide is designed to accompany the Algebra 2 Summer Work Packet. Its purpose is to offer a review of the nine specific concepts covered in the Summer Work Packet. The mastery of these concepts prior to beginning Algebra 2 is essential. For each concept, there are explanations and examples as well as extra problems that students may choose to do on their own if they are experiencing difficulty and would like to reassure themselves that they have indeed mastered the concepts. Students are not required to print out this Guide and may use it simply as an online reference as they complete their summer work. However, printing this packet with the intention of including it in their Algebra 2 binder as a reference guide would serve them well throughout the entire course.

Topics Covered in this Packet:
- A. Order of Operations
- B. Fractions
- C. Simplifying Expressions
- D. Solving Equations
- E. Solving Inequalities
- F. Linear Graphs
- G. System of Equations
- H. Multiplying, Factoring and Solving Polynomial Expressions and Equations
- I. Relations & Functions

A. Order of Operations

The rules for Order of Operations are as follows:

FIRST: Perform operations inside grouping symbols. Grouping symbols include parentheses ( ), brackets [ ] , braces { } , radical symbols , absolute value symbols and fraction bars. If an expression contains more than one set of grouping symbols, simplify the expression inside the innermost set first. Follow the order of operations within that set of grouping symbols and then work outward.

SECOND: Simplify exponents.

THIRD: Perform multiplication and division from left to right. (Remember that a fraction bar also indicates division.)

FOURTH: Perform addition and subtraction from left to right.

Hint: You can use the well-known phrase "Please Excuse My Dear Aunt Sally" to help you remember the Order of Operations. (Remember, however, that multiplication and division must be done in the order that they appear if they do not appear in parentheses. This is also true for addition and subtraction.)

Please	Parentheses
Excuse	Exponents
My	Multiplication
Dear	Division
Aunt	Addition
Sally	Subtraction

Example 1
Simplify.

$-4^2 + 24 \div 3 \cdot 2$ (Note that there are no grouping symbols. Therefore the exponent only applies to the "4" and not the "-".)

$-16 + 24 \div 3 \cdot 2$

$-16 + 8 \cdot 2$

$-16 + 16$

0

Example 2
Simplify.

$$4\left[25-(5-2)^2\right]$$

$$4\left[25-(3)^2\right]$$

$$4\left[25-9\right]$$

$$4\cdot16$$

$$64$$

Example 3
Simplify.

$$\frac{-22-2^2}{5-3}$$

$$\frac{\left(-22-2^2\right)}{(5-3)}$$ (Note that the fraction bar acts as a grouping symbol. You must simplify the numerator and denominator before dividing.)

$$\frac{-22-4}{2}$$

$$\frac{-26}{2}$$

$$-13$$

Try These:
Simplify.

1. $8\div\dfrac{1}{2}\cdot3$

2. $-20\div\left[-2(4+1)\right]$

3. $\dfrac{5+2(-8)}{(-2)^3-3}$

4. $3\sqrt{50-1}$

5. $|4-7|^2\div(-3)$

6. $\dfrac{2(-4)+22}{4^2-9}$

B. Fractions

<u>Adding & Subtracting Fractions</u>
To add or subtract fractions, you must always have a common denominator. Once the denominators are the same, you add or subtract only the numerator to get your final answer. The common denominator of choice is the Lowest Common Denominator. If you make an effort to keep the numbers in fractions as small as possible, it will make subsequent calculations much easier.

Example 1
Perform the indicated operation.

$\dfrac{3}{4} - \dfrac{8}{16}$ (Note that in this case it is going to be much easier to have a common denominator of "4" rather than "32", "48" or "64".)

$= \dfrac{3}{4} - \dfrac{2}{4}$

$= \dfrac{1}{4}$

Example 2
Perform the indicated operation.

$\dfrac{5}{6} + \dfrac{3}{8}$

$= \dfrac{20}{24} + \dfrac{9}{24}$

$= \dfrac{29}{24}$

Try These:
Perform the indicated operation.

1. $\dfrac{8}{9} - \dfrac{4}{5}$

2. $\dfrac{3}{4} + \dfrac{1}{6}$

3. $\dfrac{7}{10} - \dfrac{3}{8}$

4. $\left(\dfrac{3}{4} - \dfrac{1}{3} \right) + \dfrac{7}{12}$

5. $\dfrac{1}{2} + \dfrac{3}{5} + \dfrac{1}{10}$

6. $\dfrac{1}{5} + \dfrac{1}{6} - \dfrac{7}{15}$

<u>Multiplying Fractions</u>
Unlike adding and subtracting, you do not need a common denominator to multiply fractions. To multiply fractions you multiply the numerators and then the denominators. Then, it is good practice to always reduce your final answer. When multiplying a fraction by a whole number, you can rewrite the whole number in fraction form by putting a "1" in the denominator.

Example 3
Perform the indicated operation.

$$\frac{3}{4} \times \frac{2}{5}$$

$$= \frac{3 \times 2}{4 \times 5}$$

$$= \frac{6}{20}$$

$$= \frac{3}{10}$$

Example 4
Perform the indicated operation.

$$4 \times \frac{3}{8}$$

$$= \frac{4}{1} \times \frac{3}{8}$$

$$= \frac{12}{8}$$

$$= \frac{3}{2}$$

Try These: Perform the indicated operation.

1. $\dfrac{1}{2} \times \dfrac{4}{7}$

2. $\dfrac{7}{12} \times \dfrac{3}{4}$

3. $5\left(\dfrac{1}{3} + \dfrac{2}{5}\right)$

4. $2\left(\dfrac{3}{4}\right)\left(\dfrac{2}{7}\right)$

5. $\dfrac{4}{3} + \dfrac{2}{3} \times 4$

6. $\dfrac{3}{4} \times 8$

<u>Dividing Fractions</u>

You have heard this a thousand times: "Dividing by a fraction is the same as multiplying by its reciprocal." To get the reciprocal of a fraction, you switch the numerator and the denominator. Another way of thinking of it is: a fraction multiplied by its reciprocal will always give you "1". For example the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$, and $\frac{3}{5} \times \frac{5}{3} = 1$. As always, to maintain good form, you must reduce your final answer to its simplest form.

Example 1
Perform the indicated operation.

$\frac{3}{4} \div \frac{2}{3}$

$= \frac{3}{4} \times \frac{3}{2}$

$= \frac{9}{8}$

Example 2
Perform the indicated operation.

$\frac{2}{3} \div 6$

$= \frac{2}{3} \times \frac{1}{6}$

$= \frac{1}{9}$

Try These: Perform the indicated operation.

1. $\frac{11}{18} \div \frac{1}{3}$

2. $\frac{2}{9} \div \frac{6}{7}$

3. $\frac{9}{10} \div \frac{6}{7}$

4. $\left(\frac{3}{7} \div \frac{3}{5} \right) \div \frac{4}{7}$

5. $\frac{4}{5} \times \frac{2}{3} \div \frac{8}{5}$

6. $6 \left(\frac{11}{18} \div \frac{1}{3} \right)$

C. Simplifying Expressions

To simplify algebraic expressions you need to apply the rules for the Order of Operations and collect like terms. Like terms are terms that contain the same variables raised to the same powers. Constants (numbers with no variable) are also like terms and can be simplified according to the order of operations. Typically, simplified expressions are written with the variables in descending order according to their exponents.

Distributive Property: $a(b + c) = ab + ac$ ex. $3(x - 4) = 3x - 12$

When simplifying algebraic expressions it is important to remember the Algebraic Properties of Equality. The Distributive Property is important and is also the property with which many students have difficulties. Remember that when the coefficient in front of parentheses is preceded by a negative, that negative must also be distributed through the parentheses. For example if we apply the distributive property to $x + 4(x - 3)$, the 4 is going to be distributed through the parentheses to get $x + 4x - 12$ or $5x - 12$. However if we apply the distributive property to $x - 4(x - 3)$, the -4 must be distributed to get $x - 4x - 12$ or $-3x - 12$.

Example 1
Simplify.
$$4x(1-x) = (3x^2 - 2x + 7)$$
$$4x - 4x^2 - 3x^2 + 2x - 7$$
$$= 6x - 2x^2 - 7$$
$$= -2x^2 + 6x - 7$$

Example 2
Simplify.
$$(x - 2y) - 2(5x - y - 7)$$
$$= x - 2y - 10x + 2y + 14$$
$$= -9x + 14$$

Example 3
Simplify.
$$\frac{-12x^7 - 8x^5 + 24x^4 - 32x^2}{4x^2}$$
$$= -3x^5 - 2x^3 + 6x^2 - 8$$

Try These: Simplify.

1. $5(x+3)-7$

3. $\dfrac{\dfrac{2}{5}x^3 + \dfrac{1}{5}x^3 - \dfrac{1}{2}(3x^2 - 6x^2)}{10x}$

5. 5. $4[3(x+9)+2]-7x$

2. $12x+8x+y-7x$

4. $24x+y^2+3x+2y^2$

6. $\dfrac{1}{2}\left[(10+x)-(-6+3x)\right]$

D. Solving Equations

To solve equations, we must isolate the variable in the equation to find the value of the variable that makes the equation true. We can think of an equation as a balanced scale. We must perform the same operation to both sides of the equation to keep it equal. When solving equations remember: "What you do to one side, you must do to the other." When solving linear equations, $0 = 0$ is an acceptable answer and is translated to mean that "x" can be any real number.

Example 1
Solve for x.
$2(2x-1)=3x+4$
$4x-2=3x+4$
$4x-3x = 4+2$
$x=6$

Example 2
Solve for x.
$\dfrac{2x+3}{5} = \dfrac{x-1}{3}$ This is a proportion and can be solved by cross-multiplying.
$3(2x +3)=5(x-1)$
$6x+9=5x-5$
$6x-5x = -5-9$
$x=14$

Example 3
Solve for x.
$4x^2 = 36$
$x^2 = 9$ Remember that there are two answers here. Since "x" is squared, it's value can be both positive and negative.
$x=\pm 3$

Try These:
Solve for *x*.

1. $6-7(x+1)=-3(2-x)$

2. $-5-7-3x=-x-2(x+6)$

3. $\dfrac{2}{3}x+\dfrac{1}{2}=\dfrac{3}{5}x-\dfrac{5}{6}$

4. $\dfrac{3x-5}{4}=\dfrac{x-5}{3}$

5. $4(3x+1)-7x=6+5x-2$

6. $4x+2\left[4-2(x+2)\right]=2x-4$

E. Solving Inequalities

An inequality is a statement that two or more quantities are not equal. The statement gives information about how the value of one quantity is related to the other(s). Solving an inequality means finding the set of values for "x" that makes the statement true. Remember that when you divide both sides of an inequality by a negative, you must reverse the inequality sign.

$$\text{ex. } 3 < 5, \text{ but } 3(-2) > 5(-2)$$

Thus it follows that in solving this inequality, you must reverse the sign when you isolate the "*x*":

$3-x>5$

$-x>5-3$

$-x>2$

Therefore, $x<-2$

Compound Inequalities are used to describe the union (OR) or intersection(AND) between three values. To solve compound inequalities, you must rewrite them as simple inequalities and then solve each "piece".

$$\text{ex. } 4\le x+2\le 8$$

This compound inequality can be written as two simple inequalities making them easier to solve.

Case 1: $x+2\ge 4$ Case 2: $x+2\le 8$

           $x\ge 2$ $x\le 6$

According to these answers, x must be between 2 and 6, so the solution is:

$$2\le x\le 6$$

Example 1
Solve for x.

$4x\ge 7x+6$

$4x-7x\ge 6$

$-3x\ge 6$ Don't forget that when we divide by -3, we will have to reverse the sign.

x ≤ -2

Example 2
Solve for x.

$5\le 2x+3<9$ This is a compound inequality, so we must consider two cases:

Case 1: $2x+3\ge 5$ Case 2: $2x+3<9$

           $x\ge 1$ $x\le 3$

$1\le x<3$ Put these two solutions together:

Try These:
Solve for x.

1. $2(x-5) < -3x$

2. $-2(x-7)-4-x < 8x+32$

3. $-3(2x-4) \geq 2(5-3x)$

4. $1 \leq 2x-5 \leq 7$

5. $5 < 3x-1 < 17$

6. $5 \leq 4x-3 \leq 9$

F. Linear Graphs

Slope of a Line: The rate of change that compares the amount of change in a dependent variable (y) to the amount of change in an independent variable (x).

$$slope = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

A Few Things to Remember:
- Two lines that are parallel have the same slope.
- Two lines that are perpendicular have slopes that are negative reciprocals.
- The slope of a horizontal line is 0.
- The slope of a vertical line is undefined.
- Slope-Intercept for is y = mx + b.
- Standard form is Ax + By = C.

Example 1

Given the two points M(4, 2) and N(-2, -1) on a line, find the slope of the line $\overleftrightarrow{MN}$.

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-(-1)}{4-(-2)}$$

$$= \frac{1}{2}$$

<u>Intercepts:</u>
The *x*-intercept is the point at which the graph crosses the *x*-axis. (In other words, it's the point at which $y = 0$.) The y-intercept is the point at which the graph crosses the *y*-axis. (In other words, it's the point at which $x = 0$.) Using intercepts is one of the easiest ways to graph a line on the coordinate plane.

ex. Let's say you want to graph the line defined by:
$$3x - 2y = 12$$
To find the x-intercept, set $y = 0$ and solve for *x*:
$$3x - 2(0) = 12$$
$$3x = 12$$
$$x = 4$$
Now, on the coordinate plane, go to 4 on the *x*-axis to graph the point at which this graph crosses the *x*-axis.

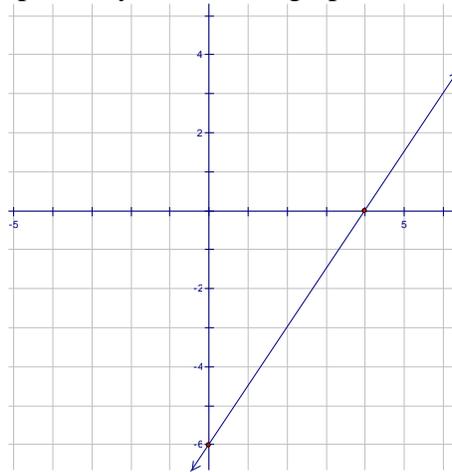To find the *y*-intercept, set $x = 0$ and solve for *y*:
$$3(0) - 2y = 12$$
$$-2y = 12$$
$$y = -6$$
Now, on the coordinate plane, go to -6 on the *y*-axis to graph the point at which this graph crosses the *y*-axis.

Connect the *x*- and *y*-intercepts and you have the graph of the line $3x - 2y = 12$

Example 2

Find the intercepts for the line $\dfrac{1}{2}x = 4y + 4$

x-int.

$\dfrac{1}{2}x = 4(0) + 4$

$\dfrac{1}{2}x = 4$

$x = 8$

x-int:$(8, 0)$

y-int.

$\dfrac{1}{2}(0) = 4y + 4$

$0 = 4y + 4$

$y = -1$

y-int:$(0, -1)$

Slope-Intercept Form
The Slope-Intercept form of the equation of a line is:
$$y = mx + b$$
Writing the equation of a line in this form gives you a lot of valuable information immediately because in this equation m is the slope of the line and b is the y-intercept.

Try These:
Find the slope of the line containing the two given points. Also, state the slope of a line that is perpendicular.

1. M(-1, 4), N(-1, -3)
2. M(0, 2), N(4, -1)
3. M(-6, 3), N(1, -4)

Find the slope as well as the x- and y-intercepts of the given lines.

4. $y = 4 - 2x$
5. $-2x - y = 4$
6. $x - 3y = -1$

G. System of Equation

A system of equations is a collection of two or more equations with a same set of unknowns. In solving a system of equations, we try to find values for each of the unknowns that will satisfy every equation in the system. The equations in the system can be linear or non-linear. This tutorial reviews systems of linear equations.

There are three specific methods to solve a system of equation: a) graphing, b) substitution, and c) linear combination/elimination.

Graphing:

The graph given below represents the system of equations, $x + y = 4$ and $x - y = -2$.
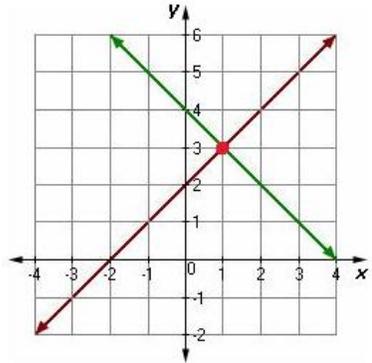
The solution would be (1, 3).

Try these:
Solve each system by graphing each equation to determine the point of intersection.

1. $y = 5x - 3$
    $y = 3x + 1$

2. $y = 2x - 13$
    $y = x - 9$

Substitution:

Given equations $3x + 2y = 7$ and $y = 2x - 7$, substitute one equation into the other, then solve.

$3x + 2(2x - 7) = 7$

$3x + 4x - 14 = 7$

$7x = 21$

$x = 3$

Then substitute this found variable into the one of the initial equations.

$y = 2(3) - 7 = -1$

The solution would be (3, -1).

Try these:
Solve each system by substitutino to determine the solution set.

1. $y = x + 1$
    $2x + y = 10$

2. $y = 2x - 1$
    $y = x + 2$

Combination/Elimination
An approach to solving system of equations that "eliminates" one variable when you combine equations.

$5x - 6y = -32$
$3x + 6y = 48$

When you combine both of these equations together the "x" terms cancel themselves out. To cancel, terms have to be opposites.

$8x = 16$

$x = 2$

Then substitute this found variable into the one of the initial equations.

$5(2) - 6y = -32$

$10 - 6y = -32$

$-6y = -42$

$y = -7$

The solution would be (2, -7).

If the terms are not opposites, then you need to create them by multiplying both sides of one or both equations by a number.

$6x + 4y = 42$
$-3x + 3y = -6$

If you multiply both sides of the second by 2, this would result in the following:

$6x + 4y = 42$
$-6x + 6y = -12$

This would result in a solution of (5, 3).

Try These:
Solve the system of equations by using the combination/elimination method.

1. $4x - 7y = 3$
    $x - 7y = -15$

2. $5x - 9y = -43$
    $3x + 8y = 68$

H. Polynomial Expressions & Equations

A polynomial is defined as a monomial or a sum or difference of monomials.

F.O.I.L. (First, Outer, Inner, Last)

FOIL is the method by which two binomials can be multiplied using the distributive property.

Example 1 Multiply the binomials using FOIL.

$(x+3)(x+2)$

Take the first term of the first binomial and distribute it through the second binomial.
Then, take the second term in the first binomial and distribute it through the second binomial.

$x^2 + 2x + 3x + 6$

$x^2 + 5x + 6$

Example 2
Multiply the binomials using FOIL.

$(x-8)(7x+4)$

$= 7x^2 + 4x - 56x - 32$

$= 7x^2 - 52x - 32$

Factoring:
A polynomial is in its factored form when it is written as a product of monomials and polynomials that cannot be factored further. There are several different types of factoring:

Greatest Common Factor:
Take out the largest factor that is common in each term and leave what is left of each term in parentheses.

Example 3
Factor.

$8x^4 + 4x^3 - 2x^2$ $2x^2$ is the largest factor that is common to each term.

$= 2x^2\left(4x^2 + 2x - 1\right)$ Use the distributive property to check this answer.

Factoring Quadratic Trinomials (x^2+bx+c):
When factoring quadratic trinomials with no coefficient in front of the x^2, ask yourself this question:
What two numbers add to give "b" and multiply to give "c"?

Example 4
Factor.
$x^2 + 7x + 12$
What two numbers add to give "7" and multiply to give "12"?
Obviously, the answer to this question is 3 and 4.
Therefore, the factors of $x^2 + 7x + 12$ are $(x + 3)(x + 4)$.

Example 5
Factor.
$x^2 - 10x + 16$
What two numbers add to give "-10" and multiply to give "16"?
The answer to this question is -2 and -8.
Therefore, the factors of $x^2 - 10x + 16$ are $(x - 2)(x - 8)$.

Example 6 Factor.
$x^2 + 7x - 18$
What two numbers add to give "7" and multiply to give "-18"?
The answer to this question is 9 and -2.
Therefore, the factors of $x^2 + 7x - 18$ are $(x + 9)(x - 2)$.

Example 7
Factor.
$x^2 - 5x - 24$
What two numbers add to give "-5" and multiply to give "-24"?
The answer to this question is -8 and 3.
Therefore, the factors of $x^2 - 5x - 24$ are $(x - 8)(x + 3)$.

Difference of Two Squares:
A polynomial is a difference of two squares if:
- There are two terms, one subtracted from the other.
- Both terms are perfect squares.

A polynomial that is a difference of two squares has the form:
$$a^2 - b^2 = (a - b)(a + b)$$

Example 8
Factor.
$4x^2 - 25$ In this case, a = 2x and b = 5.
$= (2x - 5)(2x + 5)$

Zero Product Property:
The Zero Product Property states that if the product of two quantities is zero, then at least one of the quantities equals zero. This property is particularly important when solving polynomial equations.

If $ab = 0$, then $a = 0$ or $b = 0$.

The Zero Product Property can be applied when trying to solve equations such as:

$x^2 + 7x + 10 = 0$ Factoring this trinomial will give us the form ab = 0.

$(x + 2)(x + 5) = 0$ According to the Zero Product Property, we can separate these factors.

$$x + 2 = 0 \qquad\qquad\qquad\qquad x + 5 = 0$$
$$x = -2 \qquad\qquad\qquad\qquad x = -5$$

Example 9
Solve for x.

$x^2 = 16$ Rewrite this so that the equation equals zero.

$x^2 - 16 = 0$ This is a difference of squares that can be easily factored.

$(x + 4)(x - 4) = 0$

$x = -4$ or $x = 4$

The Quadratic Formula:

The Quadratic Formula is the only method that can be used to solve any quadratic equation. To find the value of "x" in a quadratic of the form $ax^2 + bx + c = 0$, use the following:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 10 Solve using the quadratic formula.

$2x^2 = x - 3$ We need to rewrite this to put it in the form $ax^2 + bx + c = 0$.

$x^2 - 2x - 3 = 0$

$a = 1$

$b = -2$

$c = -3$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - (-12)}}{2}$$

$$x = \frac{2 \pm \sqrt{16}}{2}$$

$$x = \frac{2 \pm 4}{2}$$

$$x = \frac{2 + 4}{2} \qquad and \qquad x = \frac{2 - 4}{2}$$

$$x = 3 \qquad\qquad\qquad\qquad x = -1$$

I. Relations & Functions

A *relation* is a set of ordered pairs. The *domain* of a relation is the set of first coordinates (or *x*-values) of the ordered pairs. The *range* of a relation is the set of second coordinates (or *y*-values) of the ordered pairs.

ie. An example of a relation is $A = \{(1,5), (2,3), (3,2), (4,1)\}$

domain of $A = \{1, 2,3,4\}$
range of $A = \{5,3, 2,1\}$

A *function* is a special type of relation that pairs each domain value with exactly one range value.

Example 1:
Is the following relation a function?

$Y = \{(-4, 2), (-8, 2), (4,1), (5,1)\}$ Is every x-value paired with only one y-value?
Yes *Y* is a function.

Example 2: Is the following relation a function?
$X = \{(3,-2), (5,-1), (4,0), (3,1)\}$ Is every x-value only used once?
No, *X* is not a function because 3 is paired with two different y values (-2, & 1).

Function Notation illustrates another way to show the dependence of one variable on another. The notation $f(x)$ represents "*f* as a function of *x*".

For example, if you have the function $f(x) = 2x + 1$ and you need to determine $f(2)$. Insert the value of 2 into the function where ever you have an *x*. So $f(2) = 2(2) + 1 = 5$, so $f(2) = 5$.

Try These:
Determine the following if $g(x) = 2x^2 - 3x + 4$
1. $g(2)$
2. $g(-3)$
3. Determine *x* so that $g(x) = 9$