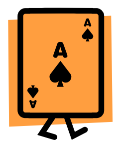
BRUSHING UP on
ESSENTIAL ALGEBRA SKILLS

To Get you Ready to



PRE- CALCULUS

Name: ______________________________

Directions: Use **pencil** and the space provided next to the question to **show all work**. The purpose of this packet is to give you a review of basic skills. Please **refrain from using a calculator!**

Prepared by:
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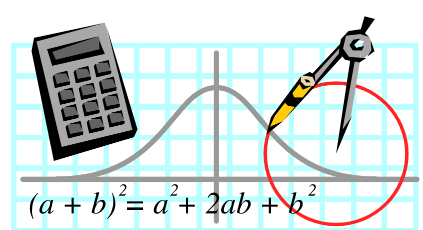

BRUSHING UP ON BASIC ALGEBRA SKILLS

Mrs. Trebat

Name: ________________________________ **DUE DATE:** __________

Directions: *Use pencil, show work, box in your answers.*

Monomial Factors of Polynomials

A **monomial** is an expression that is either a numeral, a variable, or the product of a numeral and one or more variables. Example of monomials: $7,\ x,\ 6x^2y^3$.

A sum of monomials is called a **polynomial**. Some polynomials have special names:

Binomials (two terms): $3x-5$ *or* $2xy+x^2$

Trinomials (3 terms): $x^2+5x-15$ *or* $x^2-6xy+9y^2$

Divide:

1) $\dfrac{24x-12}{6}$

2) $\dfrac{8x^4-4x^3-6x^2}{-2x^2}$

Factor (monomials only): Example: $15a-25b+35=5(3a-5b+7)$

3) $7a^3-21a^2-14a$

4) $5ax^2-10a^2x+15a^3$

Simplify: Example:

$$\frac{14x-21}{7}-\frac{10x-25}{5}=\frac{7(2x-3)}{7}-\frac{5(2x-5)}{5}=2x-3-2x+5=2$$

Factor

Cancel out common factors
Simplify and combine

5) $\dfrac{6a+9b}{3}-\dfrac{7a+21b}{7}=$

6) $\dfrac{x^2y-3x^2y^2}{xy}+\dfrac{6xy+9xy^2}{3y}=$

Multiplying Binomials Mentally

Write each product as a trinomial:

7) $(x-9)(x+4)=$

8) $(4-x)(1-x)=$

9) $(2a+5)(a-2)=$

10) $(2x-5)(3x+4)=$

Find the values of p, q, and r that make the equation true.

11) $(px+q)(2x+5)=6x^2+11x+r$

Difference of Two Squares

You must use the shortcut below (**do not "FOIL"**!!!!)

$$(a+b)(a-b)=a^2-b^2$$

$$(First+Second)\times(First-Second)=(First)^2-(Second)^2$$

Write each product as a binomial:

12) $(x+7)(x-7)=$

13) $(y+8)(y-8)=$

14) $(5x+2)(5x-2)=$

15) $(8x-11)(8x+11)=$

16) $(4a+5b)(4a-5b)=$

17) $(x^2-9y)(x^2+9y)=$

Now let's try reversing the process above... Factor:

18) $x^2-36=$

19) $m^2-81=$

20) $25a^2-1=$

21) $49x^2-9y^2=$

Factor each expression as the difference of two squares. Then simplify.

Example: $x^2-(x-3)^2=[x-(x-3)][x+(x-3)]=3(2x-3)$

Apply the formula   simplify

22) $(x+4)^2-x^2=$

23) $9(x+1)^2-4(x-1)^2=$

Squares of Binomials and Perfect Square Trinomials

Every time you square a binomial, the same pattern comes up. To speed up the process, we should memorize:

$$(a+b)^2 = a^2 + 2ab + b^2 \quad \text{and} \quad (a-b)^2 = a^2 - 2ab + b^2$$

A trinomials is called a perfect square trinomial if it is the square of a binomial. For example, $x^2 - 6x + 9$ is a perfect square trinomial because it is equal to $(x-3)^2$.

Write each square as a trinomial.

24) $(a-9)^2 =$

25) $(x+7)^2 =$

26) $(4x-1)^2 =$

27) $(5a-2b)^2 =$

Decide if each trinomial is a perfect square. If it is, factor it. Otherwise, write **not a perfect square.**

28) $a^2 + 6a + 9 =$

29) $y^2 - 14y + 49 =$

30) $121 - 22x + x^2 =$

31) $9a^2 + 30ab + 100b^2 =$

32) $49x^2 - 28xy + 4y^2 =$

33) $25x^2 - 15xy + 36y^2 =$

34) $a^2b^2 - 12ab + 36 =$

35) $121 - 33x^2 + 9x^4$

36) Show that $a^4 - 8a^2 + 16$ can be factored as $(a+2)^2(a-2)^2$.

37) Solve and check: $(x+2)^2 - (x-3)^2 = 35$

Factoring Quadratic Trinomials

To factor a trinomial of the form $x^2 + bx + c$, you must find two numbers, **r** and **s**, whose product is **c** and whose sum is **b**.

$$x^2 + bx + c = (x + r)(x + s)$$

When you find the product $(x + r)(x + s)$ you obtain

$$x^2 + bx + c = x^2 + (r + s)x + rs$$

Example: Factor $x^2 - 2x - 15$.

a. List the factors of -15 (the last term).
b. Either write them down or do this mentally. Find the pairs of factors with sum -2 (the middle term).

Factors of -15	Sum of the factors
1, -15	-15 (*discard)*
-3, 5	2 (discard)
3, -5	**-2** (keep)

$\therefore x^2 - 2x - 15 = (x - 5)(x + 3)$

Check the result by multiplying…

Factor. Check by multiplying (mentally).

38. $x^2 + 8x + 12 =$ ____________________
39. $x^2 - 7x + 12 =$ ____________________
40. $x^2 - 4x + 12 =$ ____________________
41. $x^2 - 9x + 18 =$ ____________________
42. $x^2 - 3x - 18 =$ ____________________
43. $x^2 + 11x + 18 =$ __________________
44. $x^2 - 5x - 36 =$ __________________
45. $x^2 - 15x + 36 =$ ________________
46. $x^2 - 9x - 36 =$ __________________
47. $x^2 + 3x - 28 =$ __________________

Factoring General Quadratic Trinomials of the type $ax^2 + bx + c$

Example: Factor $2x^2 + 7x - 9 = (2x+9)(x-1)$

List all factors Of $2x^2$ — **List factors of -9**

Factor:

48) $3x^2 + 7x + 2$

49) $2x^2 + 5x + 3$

50) $2x^2 - 15x + 7$

51) $3a^2 + 4a - 4$

52) $5a^2 - 6a - 2$

53) $3x^2 - 2x - 5$

54) $3m^2 + 7m - 6$

55) $4a^2 - a - 3$

Factor by Grouping

Example 1: $7(a-2) + 3a(a-2) = (7+3a)(a-2)$

*Notice that we "**factored out**" the common factor (a - 2)*

Example 2: $5(x-3) - 2x(3-x)$

Notice that $x-3$ *and* $3-x$ *are opposites.*

$$5(x-3) - 2x(3-x) = 5(x-3) + 2x(x-3) = (5+2x)(x-3)$$

56) $2x(x-y) + y(y-x)$

57) $3a(2b-a) - 2b(a-2b)$

Group and Factor:

58) $3a + ab + 3c + bc$

59) $3a^3 + a^2 + 6a + 2$

60) $3ab - b - 4 + 12a$

Solving Equations by Factoring – The Zero Product Property

Key Concept:

Zero-Product Property For all real numbers a and b: $a \cdot b = 0$ if and only if $a = 0$ or $b = 0$ A product of factors is equal to zero if and only if at least one of the factors is 0.

Solve:

61) $(x+5)(x-3)=0$

62) $2x(x-9)=0$

63) $(2a-3)(3a+2)=0$

64) $3x(5x+2)(x-7)=0$

65) $a^2-3a+2=0$

66) $x^2-12x+35=0$

67) $b^2=4b+32$

68) $25x^2-16=0$

69) $7x^2=18x-11$

70) $8y^3-2y^2=0$

71) $4x^3-12x^2+8x=0$

72) $9x^3+25x=30x^2$

Sample for items 73-74: $(a-1)(a+3)=12$

$a^2+2a-3-12=0$ expand the left side; bring over 12; set it = 0;

$(a-3)(a+5)=0$ combine like terms; factor it;

$a=3 \text{ or } a=-5$

73) $(x+1)(x-5)=16$

74) $(2z-5)(z-1)=2$

Simplifying Fractions – Follow the 2 examples below.

Example 1: Simplify $\frac{3x+6}{3x+3y}$.

Solution: $\frac{3x+6}{3x+3y}=\frac{3(x+2)}{3(x+y)}$ Factor the numerator and denominator; look for common factors;

$=\frac{x+2}{x+y},\ x\neq -y$ Cancel out common factor which is 3;

Example 2: Simplify $\frac{x^2-9}{(2x+1)(3-x)}$

Solution: $\frac{x^2-9}{(2x+1)(3-x)}=\frac{(x+3)(x-3)}{-(2x+1)(x-3)}$ First factor the numerator; "pull out" a negative in the factor (3-x) to make it a common factor with the numerator;

$=-\frac{x+3}{2x+1},\left(x\neq-\frac{1}{2},x\neq 3\right)$ exclude the first as it would make the denominator =0; exclude the 2nd as it would make both numerator and denominator =0.

Simplify. Give any restrictions on the variable.

75) $\frac{5x-10}{x-2}$

76) $\frac{2a-4}{a^2-4}$

77) $\frac{(x-5)(2+7x)}{(5-x)(7x+2)}$

78) $\frac{x^2+8x+16}{16-x^2}$

Multiplying Fractions

Multiplication Rule for Fractions

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$ To multiply fractions, you multiply their numerators and multiply their denominators.

Note: You can multiply first and then simplify, or you can simplify first and then multiply.

Example: $$\frac{x^2-x-12}{x^2-5x}\cdot\frac{x^2-25}{x+3}=\frac{(x-4)\cancel{(x+3)}}{x\cancel{(x-5)}}\cdot\frac{\cancel{(x-5)}(x+5)}{\cancel{x+3}}$$

$$=\frac{(x-4)(x+5)}{x}$$ Notice how common factors were cancelled.

Simplify.

79) $\frac{a+2}{a^2}\cdot\frac{3a}{a^2-4}$

80) $\frac{a^2-x^2}{a^2}\cdot\frac{a}{3x-3a}$

81) A triangle has base $\frac{3x}{4}$ cm and height $\frac{8}{9x}$ cm. What is its area?

Dividing Fractions

Division Rule for Fractions: $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$ *Example*: $\frac{7}{9}\div\frac{2}{3}=\frac{7}{9}\cdot\frac{3}{2}=\frac{7}{6}$

To divide by a fraction, you multiply by its reciprocal!

82) $\frac{2+2a}{6} \div \frac{1+a}{9} =$

83) $\frac{x^2-1}{2} \div \frac{x+1}{16} =$

84) $\frac{2a+2b}{a^2} \div \frac{a^2-b^2}{4a}$

85) $\frac{4x^2-25}{x^2-16} \div \frac{12x+30}{2x^2+8x} =$

Adding and Subtracting Algebraic Fractions

Key Steps: (1) Find the Least Common Denominator (**LCD**);
(2) Re-write each fraction being added or subtracted with the same common denominator.
(3) Add or subtract their numerators and write the result over the common denominator.

Example:

$$\frac{3}{6x-30} + \frac{8}{9x-45} = \frac{3}{6(x-5)} + \frac{8}{9(x-5)}$$ Factor out denominators to more easily identify LCD

$$= \frac{9}{18(x-5)} + \frac{16}{18(x-5)}$$ multiply the first fraction by 3, the second by 2

$$= \frac{25}{18(x-5)} \text{ or } \frac{25}{18x-90}$$

86) $\frac{4x+3}{3} - \frac{7x}{4} + \frac{x-3}{6} =$

87) $\frac{2}{x-3}+\frac{4}{x+3}=$

88) $\frac{x}{x^2-1}+\frac{4}{x+1}=$

89) $\frac{3a}{a-2b}+\frac{6b}{2b-a}=$

90) $\frac{x-11}{x^2-9}-\frac{x-7}{x^2-3x}=$

BRUSHING UP ON BASIC ALGEBRA SKILLS – Solved Problems

Mrs. Trebat

Name: ______________________________

Directions: *Use pencil. You must show work for credit. Your final answers must be clearly identified.*

Monomial Factors of Polynomials

A **monomial** is an expression that is either a numeral, a variable, or the product of a numeral and one or more variables. Example of monomials: $7,\ x, 6x^2y^3$.

A sum of monomials is called a **polynomial**. Some polynomials have special names:

Binomials (two terms): $3x-5$ *or* $2xy+x^2$

Trinomials (3 terms): $x^2+5x-15$ *or* $x^2-6xy+9y^2$

Divide:

1) $\dfrac{24x-12}{6} = \dfrac{12(2x-1)}{6} = 4x-2$

2) $\dfrac{8x^4-4x^3-6x^2}{-2x^2} = \dfrac{2x^2(4x^2-2x-3)}{-2x^2} = -4x^2+2x+3$

Factor: Example: $15a-25b+35 = 5(3a-5b+7)$

3) $7a^3-21a^2-14a$

$7a(a^2-3a-2)$

4) $5ax^2-10a^2x+15a^3$

$5a(x^2-2ax+3a^2)$

Simplify: Example:

$$\frac{14x-21}{7}-\frac{10x-25}{5}=\frac{7(2x-3)}{7}-\frac{5(2x-5)}{5}=2x-3-2x+5=2$$

Factor — Cancel out common factors — Simplify and combine

5) $\dfrac{6a+9b}{3}-\dfrac{7a+21b}{7} = \dfrac{3(2a+3b)}{3}-\dfrac{7(a+3b)}{7} = 2a+3b-a-3b = a$

6) $\dfrac{x^2y-3x^2y^2}{xy}+\dfrac{6xy+9xy^2}{3y} = \dfrac{xy(x-3xy)}{xy}+\dfrac{3xy(2+3y)}{3y} =$

$= x-3xy+2x+3xy$

$= 3x$

Multiplying Binomials Mentally

Write each product as a trinomial:

7) $(x-9)(x+4) = x^2-5x-36$

8) $(4-x)(1-x) = 4-5x+x^2$

9) $(2a+5)(a-2) = 2a^2+a-10$

10) $(2x-5)(3x+4) = 6x^2-7x-20$

Find the values of p, q, and r that make the equation true.

11) $(px+q)(2x+5)=6x^2+11x+r$

$2px^2+5px+2qx+5q = 6x^2+11x+r$

$2px^2+(5p+2q)x+5q = 6x^2+11x+r$

Equating like Terms:

$2p=6 \Rightarrow \boxed{p=3}$

$5p+2q=11$

$15+2q=11$

$2q=-4$

$\boxed{q=-2}$

$5(-2)=r$

$\boxed{-10=r}$

Difference of Two Squares

You must use the shortcut below (do not "FOIL"!!!!)

$$(a+b)(a-b)=a^2-b^2$$
$$(First+Second)\times(First-Second)=(First)^2-(Second)^2$$

Write each product as a binomial:

12) $(x+7)(x-7) = x^2-49$

13) $(y+8)(y-8) = y^2-64$

14) $(5x+2)(5x-2) = 25x^2-4$

15) $(8x-11)(8x+11) = 64x^2-121$

16) $(4a+5b)(4a-5b) = 16a^2-25b^2$

17) $(x^2-9y)(x^2+9y) = x^4-81y^2$

Now let's try reversing the process above... Factor:

18) $x^2-36 = (x+6)(x-6)$

19) $m^2-81 = (m+9)(m-9)$

20) $25a^2-1 = (5a+1)(5a-1)$

21) $49x^2-9y^2 = (7x+3y)(7x-3y)$

Factor each expression as the difference of two squares. Then simplify.

Example: $x^2-(x-3)^2=[x-(x-3)][x+(x-3)]=3(2x-3)$

Apply the formula — simplify

22) $(x+4)^2-x^2 =$

$=(x+4+x)(x+4-x)$

$=(2x+4)4$ or $4(2x+4)$

23) $9(x+1)^2-4(x-1)^2 =$

$=[3(x+1)+2(x-1)][3(x+1)-2(x-1)]$

$=(3x+3+2x-2)(3x+3-2x+2)$

$=(5x+1)(x+5)$

Squares of Binomials and Perfect Square Trinomials

Every time you square a binomial, the same pattern comes up. To speed up the process, we should memorize:

$$(a+b)^2 = a^2 + 2ab + b^2 \quad \text{and} \quad (a-b)^2 = a^2 - 2ab + b^2$$

A trinomials is called a perfect square trinomial if it is the square of a binomial. For example, $x^2 - 6x + 9$ is a perfect square trinomial because it is equal to $(x-3)^2$.

Write each square as a trinomial.

24) $(a-9)^2 =$ $a^2 - 18a + 81$

25) $(x+7)^2 =$ $x^2 + 14x + 49$

26) $(4x-1)^2 =$ $16x^2 - 8x + 1$

27) $(5a-2b)^2 =$ $25a^2 - 20ab + 4b^2$

Decide if each trinomial is a perfect square. If it is, factor it. Otherwise, write ***not a perfect square***.

28) $a^2 + 6a + 9 =$ $(a+3)^2$

29) $y^2 - 14y + 49 =$ $(y-7)^2$

30) $121 - 22x + x^2 =$ $(11-x)^2$

31) $9a^2 + 30ab + 100b^2 =$ NOT A PERFECT SQUARE (due to middle term...)

32) $49x^2 - 28xy + 4y^2 =$ $(7x-2y)^2$

33) $25x^2 - 15xy + 36y^2 =$ NOT A PERFECT □!

34) $a^2b^2 - 12ab + 36 =$ $(ab-6)^2$

35) $121 - 33x^2 + 9x^4$ NOT A PERFECT □!

36) Show that $a^4 - 8a^2 + 16$ can be factored as $(a+2)^2(a-2)^2$.

$a^4 - 8a^2 + 16 = (a^2-4)^2 = [(a+2)(a-2)]^2 = (a+2)^2(a-2)^2$

37) Solve and check: $(x+2)^2 - (x-3)^2 = 35$

$x^2 + 4x + 4 - (x^2 - 6x + 9) = 35$

$4x + 4 + 6x - 9 = 35$

$10x - 5 = 35$

$10x = 40$

$\boxed{x = 4}$

Factoring Quadratic Trinomials

To factor a trinomial of the form $x^2 + bx + c$, you must find two numbers, r and s, whose product is c and whose sum is b.

$$x^2 + bx + c = (x+r)(x+s)$$

When you find the product $(x+r)(x+s)$ you obtain

$$x^2 + bx + c = x^2 + (r+s)x + rs$$

Example: Factor $x^2 - 2x - 15$.

a. List the factors of -15 (the last term).
b. Either write them down or do this mentally. Find the pairs of factors with sum -2 (the middle term).

Factors of -15	Sum of the factors
1, -15	-15 *(discard)*
-3, 5	2 (discard)
3, -5	-2 (keep)

$\therefore x^2 - 2x - 15 = (x-5)(x+3)$

Check the result by multiplying...

Factor. Check by multiplying (mentally).

38. $x^2 + 8x + 12 =$ $(x+6)(x+2)$
39. $x^2 - 7x + 12 =$ $(x-4)(x-3)$
40. $x^2 - 4x + 12 =$ NOT Factorable
41. $x^2 - 9x + 18 =$ $(x-6)(x-3)$
42. $x^2 - 3x - 18 =$ $(x-6)(x+3)$
43. $x^2 + 11x + 18 =$ $(x+9)(x+2)$
44. $x^2 - 5x - 36 =$ $(x-9)(x+5)$
45. $x^2 - 15x + 36 =$ $(x-12)(x-3)$
46. $x^2 - 9x - 36 =$ $(x-12)(x+3)$
47. $x^2 + 3x - 28 =$ $(x+7)(x-4)$

Factoring General Quadratic Trinomials of the type ax^2+bx+c

Example: Factor $2x^2+7x-9 = (2x+9)(x-1)$

List all factors Of $2x^2$ — List factors of -9

Factor:

48) $3x^2+7x+2$
$(3x+1)(x+2)$

49) $2x^2+5x+3$
$(2x+3)(x+1)$

50) $2x^2-15x+7$
$(2x-1)(x-7)$

51) $3a^2+4a-4$
$(3a-2)(a+2)$

52) $5a^2-6a-2$
$(5a+1)(a-2)$

53) $3x^2-2x-5$
$(3x-5)(x+1)$

54) $3m^2+7m-6$
$(3m-2)(m+3)$

55) $4a^2-a-3$
$(4a+3)(a-1)$

Factor by Grouping

Example 1: $7(a-2)+3a(a-2)=(7+3a)(a-2)$

Notice that we "factored out" the common factor (a - 2)

Example 2: $5(x-3)-2x(3-x)$

Notice that $x-3$ and $3-x$ are opposites.

$5(x-3)-2x(3-x)=5(x-3)+2x(x-3)=(5+2x)(x-3)$

56) $2x(x-y)+y(y-x) = 2x(x-y)-y(x-y) = (x-y)(2x-y)$

57) $3a(2b-a)-2b(a-2b) = 3a(2b-a)+2b(2b-a) = (2b-a)(3a+2b)$

Group and Factor:

58) $3a+ab+3c+bc = (3a+ab)+(3c+bc) = a(3+b)+c(3+b) = (a+c)(3+b)$

59) $3a^3+a^2+6a+2 = (3a^3+a^2)+(6a+2) = a^2(3a+1)+2(3a+1) = (3a+1)(a^2+2)$

60) $3ab-b-4+12a$
$= (3ab-b)-(4-12a) = b(3a-1)-4(1-3a)$
$= b(3a-1)+4(3a-1)$
$= (3a-1)(b+4)$

Solving Equations by Factoring – The Zero Product Property

Key Concept:

Zero-Product Property
For all real numbers a and b:
$a \cdot b = 0$ if and only if $a=0$ or $b=0$
A product of factors is equal to zero if and only if at least one of the factors is 0.

Solve:

61) $(x+5)(x-3)=0$
$x+5=0$ or $x-3=0$
$x=-5$ or $x=3$

62) $2x(x-9)=0$
$2x=0 \Rightarrow x=0$
$x-9=0 \Rightarrow x=9$

63) $(2a-3)(3a+2)=0$
$a=3/2$ or $a=-2/3$

64) $3x(5x+2)(x-7)=0$
$3x=0$, $5x+2=0$, $x-7=0$
$x=0$, $x=-2/5$, $x=7$

65) $a^2-3a+2=0$
$(a-2)(a-1)=0$
$a=2$ or $a=1$

66) $x^2-12x+35=0$
$(x-7)(x-5)=0$
$x=7$ or $x=5$

67) $b^2=4b+32$
$b^2-4b-32=0$
$(b-8)(b+4)=0$
$b=8$ or $b=-4$

68) $25x^2-16=0$
$(5x-4)^2=0 \Rightarrow 5x-4=0 \Rightarrow x=\frac{4}{5}$

69) $7x^2=18x-11$
$7x^2-18x+11=0$
$(7x-11)(x-1)=0$
$x=11/7$ or $x=1$

70) $8y^3-2y^2=0$
$2y^2(4y-1)=0 \Rightarrow y=0$ or $y=\frac{1}{4}$

71) $4x^3-12x^2+8x=0$
$4x(x^2-3x+2)=0$
$4x(x-2)(x-1)=0$
$x=0$ $x=2$ $x=1$

72) $9x^3+25x=30x^2$
$9x^3-30x^2+25x=0$
$x(9x^2-30x+25)=0$
$x(3x-5)^2=0 \Rightarrow x=0$ or $x=5/3$

Sample for items 73-74: $(a-1)(a+3)=12$

$a^2+2a-3-12=0$ expand the left side; bring over 12; set it = 0;
$(a-3)(a+5)=0$ combine like terms; factor it;
$a=3$ *or* $a=-5$

73) $(x+1)(x-5)=16$
$x^2-4x-5=16$
$x^2-4x-21=0$
$(x-7)(x+3)=0$
$x=7$ or $x=-3$

74) $(2z-5)(z-1)=2$
$2z^2-7z+5=2$
$2z^2-7z+3=0$
$(2z-1)(z-3)=0$
$z=1/2$ or $z=3$

Simplifying Fractions - Follow the 2 examples below.

Example 1: Simplify $\frac{3x+6}{3x+3y}$.

Solution: $\frac{3x+6}{3x+3y} = \frac{3(x+2)}{3(x+y)}$ Factor the numerator and denominator; look for common factors;

$= \frac{x+2}{x+y},\ x \neq -y$ Cancel out common factor which is 3;

Example 2: Simplify $\frac{x^2-9}{(2x+1)(3-x)}$

Solution: $\frac{x^2-9}{(2x+1)(3-x)} = \frac{(x+3)(x-3)}{-(2x+1)(x-3)}$ First factor the numerator; "pull out" a negative in the factor (3-x) to make it a common factor with the numerator;

$= -\frac{x+3}{2x+1},\ \left(x \neq -\frac{1}{2}, x \neq 3\right)$ exclude the first as it would make the denominator =0; exclude the 2nd as it would make both numerator and denominator =0.

Simplify. Give any restrictions on the variable.

75) $\frac{5x-10}{x-2} = \frac{5(x-2)}{x-2} = 5$

76) $\frac{2a-4}{a^2-4} = \frac{2(a-2)}{(a+2)(a-2)} = \frac{2}{a+2}$

$a \neq \pm 2$ ← note!

77) $\frac{(x-5)(2+7x)}{(5-x)(7x+2)}$

$= \frac{(x-5)(2+7x)}{-(x-5)(7x+2)}$

$= -1$

$x \neq 5,\ x \neq -2/7$

78) $\frac{x^2+8x+16}{16-x^2} = \frac{(x+4)^2}{(4+x)(4-x)} =$

$= \frac{x+4}{4-x},\ x \neq \pm 4$

Multiplying Fractions

Multiplication Rule for Fractions

$\frac{a}{b}\cdot\frac{c}{d} = \frac{ac}{bd}$ To multiply fractions, you multiply their numerators and multiply their denominators.

Note: You can multiply first and then simplify, or you can simplify first and then multiply.

Example: $\frac{x^2-x-12}{x^2-5x}\cdot\frac{x^2-25}{x+3} = \frac{(x-4)(x+3)}{x(x-5)}\cdot\frac{(x-5)(x+5)}{x+3}$

$= \frac{(x-4)(x+5)}{x}$ Notice how common factors were cancelled.

Simplify.

79) $\frac{a+2}{a^2}\cdot\frac{3a}{a^2-4} =$

$= \frac{a+2}{a^2}\cdot\frac{3a}{(a+2)(a-2)} =$

$= \frac{3}{a(a-2)}$

80) $\frac{a^2-x^2}{a^2}\cdot\frac{a}{3x-3a} =$

$= \frac{(a+x)(a-x)}{a^2}\cdot\frac{a}{3(x-a)}$

$= \frac{-(a+x)(x-a)}{a}\cdot\frac{1}{3(x-a)}$

$= \boxed{\frac{-(a+x)}{3a}}$ or $\boxed{-\frac{a+x}{3a}}$

81) A triangle has base $\frac{3x}{4}$ cm and height $\frac{8}{9x}$ cm. What is its area?

Area $= \frac{1}{2}$(base)(height)

$A = \frac{1}{2}\left(\frac{3x}{4}\right)\left(\frac{8}{9x}\right) = \frac{1}{3}$

Dividing Fractions

Division Rule for Fractions: $\frac{a}{b}\div\frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c}$ *Example*: $\frac{7}{9}\div\frac{2}{3} = \frac{7}{9}\cdot\frac{3}{2} = \frac{7}{6}$

To divide by a fraction, you multiply by its reciprocal!

82) $\frac{2+2a}{6} \div \frac{1+a}{9} =$

$\frac{2(1+a)}{6} \cdot \frac{9}{1+a} = 3$

83) $\frac{x^2-1}{2} \div \frac{x+1}{16} =$

$\frac{(x+1)(x-1)}{2} \cdot \frac{16}{x+1} = 8(x-1)$

84) $\frac{2a+2b}{a^2} \div \frac{a^2-b^2}{4a}$

$= \frac{2(a+b)}{a^2} \cdot \frac{4a}{(a+b)(a-b)}$

$= \frac{8}{a(a-b)}$

85) $\frac{4x^2-25}{x^2-16} \div \frac{12x+30}{2x^2+8x} =$

$\frac{(2x+5)(2x-5)}{(x+4)(x-4)} \cdot \frac{2x(x+4)}{6(2x+5)} =$

$= \frac{x(2x-5)}{3(x-4)}$

Adding and Subtracting Algebraic Fractions

Key Steps: (1) Find the Least Common Denominator (**LCD**);
(2) Re-write each fraction being added or subtracted with the same common denominator.
(3) Add or subtract their numerators and write the result over the common denominator.

Example:

$\frac{3}{6x-30} + \frac{8}{9x-45} = \frac{3}{6(x-5)} + \frac{8}{9(x-5)}$ Factor out denominators to more easily identify LCD

$= \frac{9}{18(x-5)} + \frac{16}{18(x-5)}$ multiply the first fraction by 3, the second by 2

$= \frac{25}{18(x-5)}$ or $\frac{25}{18x-90}$

86) $\frac{4x+3}{3} - \frac{7x}{4} + \frac{x-3}{6} =$ $\frac{16x+12-21x+2x-6}{12} = \frac{-3x+6}{12} = \frac{-x+2}{4}$ or $\frac{2-x}{4}$

87) $\frac{2}{x-3} + \frac{4}{x+3} =$ $\frac{2(x+3)+4(x-3)}{(x-3)(x+3)} = \frac{2x+6+4x-12}{(x-3)(x+3)} = \frac{6x-6}{(x-3)(x+3)} =$

$= \frac{6(x-1)}{(x-3)(x+3)}$

88) $\frac{x}{x^2-1} + \frac{4}{x+1} =$ $\frac{x+4(x-1)}{x^2-1} = \frac{5x-4}{x^2-1}$

89) $\frac{3a}{a-2b} + \frac{6b}{2b-a} =$ $\frac{3a}{a-2b} - \frac{6b}{a-2b} = \frac{3a-6b}{a-2b} = \frac{3(a-2b)}{a-2b} =$

$= 3$

90) $\frac{x-11}{x^2-9} - \frac{x-7}{x^2-3x} =$ $\frac{x-11}{(x+3)(x-3)} - \frac{x-7}{x(x-3)}$

$= \frac{x(x-11)-(x-7)(x+3)}{x(x+3)(x-3)}$

$= \frac{x^2-11x-(x^2-4x-21)}{x(x+3)(x-3)} = \frac{-7x+21}{x(x+3)(x-3)}$

$= \frac{-7(x-3)}{x(x+3)(x-3)} = \frac{-7}{x(x+3)}$