

Algebra 32 Midterm Review Packet

Formulas you will receive on the Midterm:

$$y = a \cdot b^x$$

$$A = Pe^{rt}$$

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = P(1 + r)^t$$

$$A = P(1 - r)^t$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Name: _____

Teacher: _____

Day/Period: _____

Date of Midterm: _____

Functions:

- Vocabulary:
 - Domain (Input) & Range (Output)
 - Increasing & Decreasing
 - Minimum & Maximum
- Types of Functions:
 - Linear, Quadratic (parabola), Cubic, Quartic, Polynomial
 - Piecewise
 - Absolute Value
- Evaluate functions for values without a calculator by direct or synthetic substitution
- Transformations on Functions
 - Translations (left, right, up, down)
 - Reflections (over x-axis or y-axis)
 - Dilations (vertical stretch or shrink)

1) Answer the following questions about the graph $f(x)$.

a) Use interval notation:

Domain: _____

Range: _____

On what interval is $f(x)$ decreasing? _____

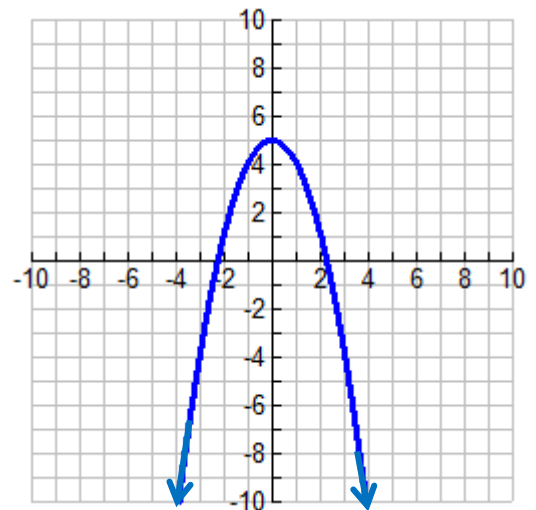
On what interval is $f(x)$ increasing? _____

b) What is the maximum? _____

c) Estimate the x-intercepts: _____ & _____

d) Find the values on $f(x)$:

- $f(1)$
- $f(0)$
- $f(x) = -4$



2) Use the function $g(x) = x^4 - 3x^2$ to answer the following questions without a calculator:

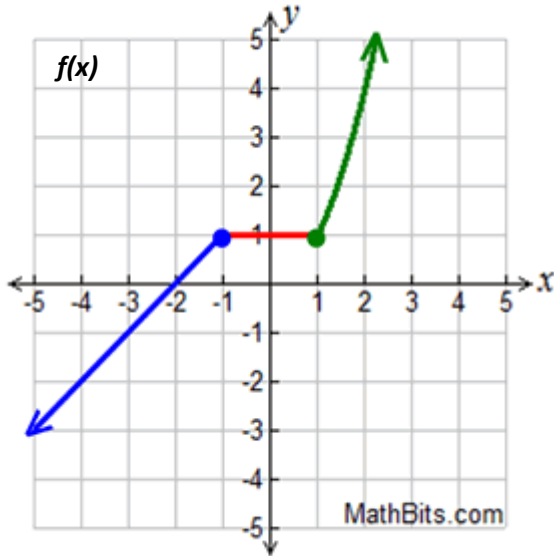
- Find the value of $g(x)$ when $x = 1$
- Find the value of $g(x)$ when $x = -1$

3) Find the value of $f(x) = -2x + 8$ when

- $x = 8$
- $f(x) = 12$
- $x = -2$
- $f(x) = 16$
- $x = -\frac{1}{2}$

- 4) Use the function $h(x) = x^3 + 2x^2 + 4x + 2$ to answer the following questions without a calculator:
- Find the value of $h(x)$ when $x = -1$
 - Find the value of $h(x)$ when $x = 2$

- 5) Answer the following questions about the piecewise function shown below.



- a) Use interval notation:

Domain: _____

Range: _____

- b) What is the y-intercept? _____

- c) What is the x-intercept? _____

- d) Estimate the following using the graph:

- $f(2)$
- $f(-1)$
- $f(x) = -2$
- $f(x) = 4$

- 6) Describe the transformations in detail: reflect over y axis, translate left 2, dilation- vertical stretch, etc...

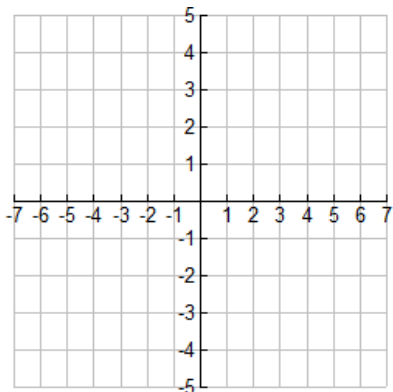
- $f(x) + 10$
- $f(-x)$
- $-f(x)$
- $3f(x)$
- $f(x + 4)$

- 7) Describe how the parent functions $g(x) = |x|$, $h(x) = x^2$ and $k(x) = \sqrt{x}$ are transformed.

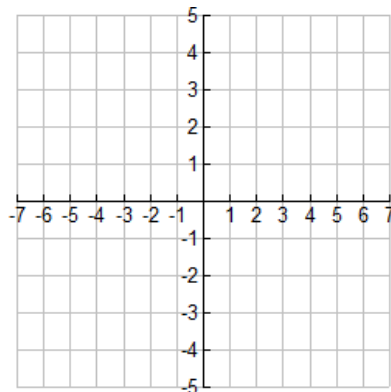
- $g(x) = |x| - 4$
- $g(x) = |x| + 6$
- $g(x) = 3|x|$
- $g(x) = -2|x|$
- $g(x) = \frac{1}{2}|x|$
- $h(x) = -(x)^2$
- $h(x) = (x)^2 - 4$
- $h(x) = 4(x)^2$
- $k(x) = \sqrt{(x + 1)}$
- $k(x) = \sqrt{x - 2}$
- $k(x) = \frac{1}{3}\sqrt{x}$

8) Graph the equations:

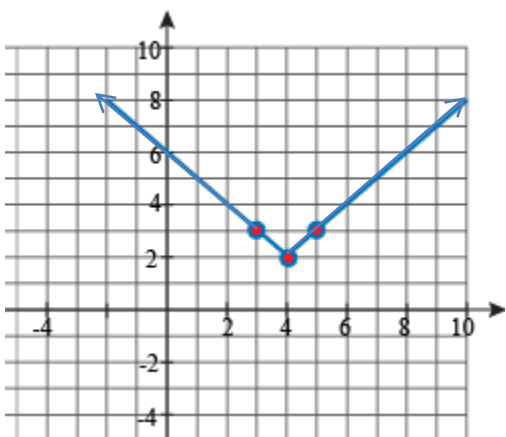
a) $y = |x - 1|$



b) $y = |x| + 3$



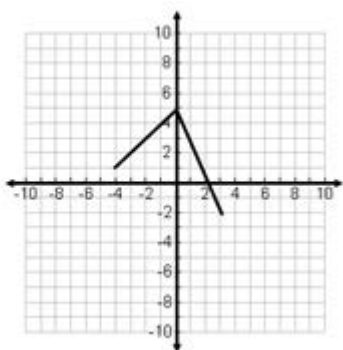
9) Which of the options is the equation for the graph shown below?



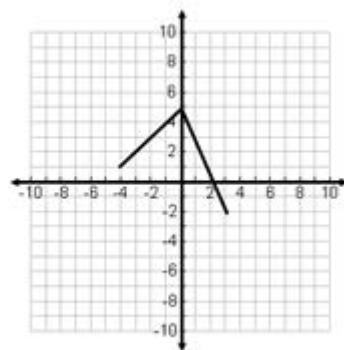
- a) $y = |x - 4| - 2$
- b) $y = |x - 4| + 2$
- c) $y = |x + 4| + 2$
- d) $y = |x + 4| - 2$

10) Perform the given transformation:

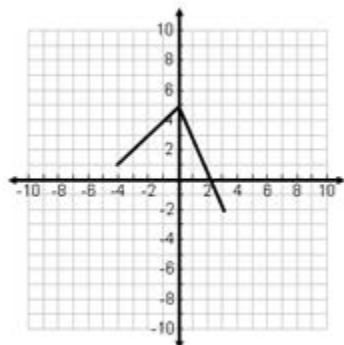
a) $-f(x)$



b) $f(x) + 1$



c) $f(x) - 5$



Polynomials:

- Vocabulary:
 - Leading Coefficient and Degree
 - Real vs. Complex Solutions
 - Zeros = x-intercepts = roots = solutions
 - y-intercept
 - end behavior
 - Standard and Factored Form
 - imaginary and complex numbers
- Skills:
 - Find values of polynomials by either direct or synthetic substitution
 - Add/Subtract/Multiply/Divide polynomials
 - Long or Synthetic Division
 - Factor polynomials:
 - Always start by looking for a GCF
 - Next, look for perfect squares
 - Lastly, use the box method or grouping where appropriate
 - Always re-check your factors to see if they can be factored further
 - Finding potential rational roots of polynomials and determining actual roots
 - Graphing Polynomials (without a calculator):
 - Finding relative min and max of a graph
 - From factored form or standard form
 - Understand end behavior and shape from degree and leading coefficient
 - In factored form: roots that touch vs. cross

11) Show all work as you perform the indicated operation:

- a) $(4x^4 + 3x^3 - 2x + 9) + (5x^3 - 2x^2 - 3)$
- b) $(8x^3 - 2x^2 + 6x) - (5x^3 - 4x^2 + 3)$
- c) $(4x + 3)(3x - 2)$
- d) $(5x + 6)^2$
- e) $(x + 2)(4x^2 - 3x + 1)$
- f) $(x - 1)(x^2 + 2x - 4)$

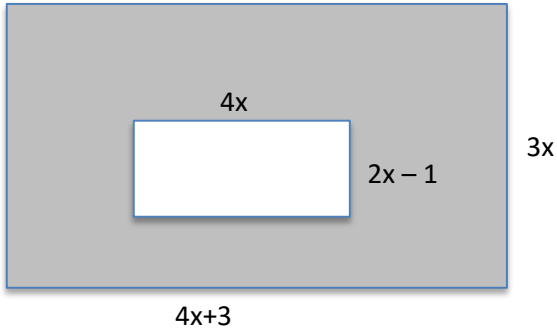
12) Factor the expressions completely:

- a) $x^2 - 1$
- b) $6x^2 - 17x + 5$
- c) $x^4 + 8x^2 + 16$
- d) $5x^4 - x^3 + 10x - 2$
- e) $x^4 + 7x^2 + 12$
- f) $x^4 - 2x^2 - 8$
- g) $3x^2 - 18$
- h) $x^3 - 2x^2 - 4x + 8$

13) Show all work as you solve the polynomials:

- a) $x^3 + x^2 - 12x = 0$
- b) $0 = 4x^3 - 8x^2 - 12x$
- c) $0 = 20x^5 - 5x^3$
- d) $3x^2 - 75 = 0$
- e) $2x^2 + 72 = 0$
- f) $x^4 - 5x^2 - 6 = 0$

14) Use the picture below to answer the questions:



- a) Write a polynomial in standard form for the area of the larger rectangle.
- b) Write a polynomial in standard form for the area of the smaller rectangle.
- c) Write a polynomial in standard form for the area of the shaded region.

15) List the possible/potential rational roots for $f(x) = 3x^5 - 9x^3 + x - 6$

16) Find all the zeros of the function $f(x) = x^3 - 3x^2 - 6x + 8$.

17) Complete the end behavior for each without using a calculator:

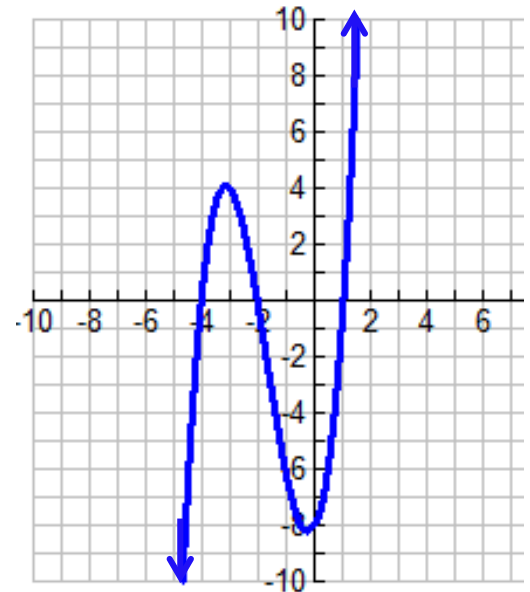
- a) $f(x) = -2x^4 + 6x + 8$
 $as\ x \rightarrow \infty\ f(x) \rightarrow \underline{\hspace{2cm}}$
 $as\ x \rightarrow -\infty\ f(x) \rightarrow \underline{\hspace{2cm}}$
- b) $f(x) = 3x^5 - 6x^4 - 8x - 10$
 $as\ x \rightarrow \infty\ f(x) \rightarrow \underline{\hspace{2cm}}$
 $as\ x \rightarrow -\infty\ f(x) \rightarrow \underline{\hspace{2cm}}$

18) The zeros of a function are $x=1$ and $x=-2$. The function does not pass through the x -axis at $x = -2$. Write one equation in factored form.

19) The zeros of a function are $x=1$, $x=-2$, and $x=3$. The function passes through the x -axis at all of these values. Write an equation in factored form.

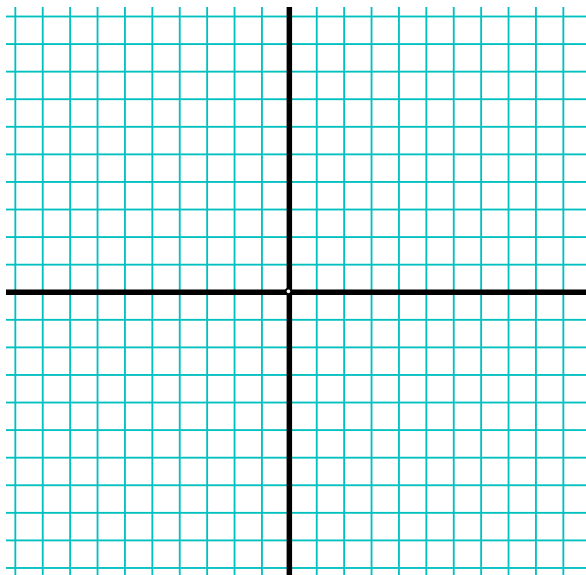
20) Use the graph below to answer the following questions:

- a) List the Zeros: _____
- b) Write a possible equation for $f(x)$: _____
- c) Is the degree odd or even?
- d) Is the leading coefficient positive or negative?
- e) Estimate the relative maximum: _____
 Estimate the relative minimum: _____
- f) Describe the End Behavior:
 as $x \rightarrow -\infty$, $f(x) \rightarrow$ _____
 as $x \rightarrow +\infty$, $f(x) \rightarrow$ _____



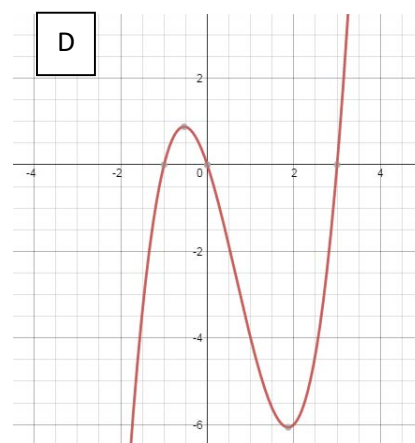
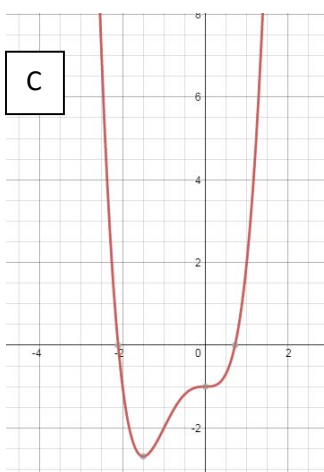
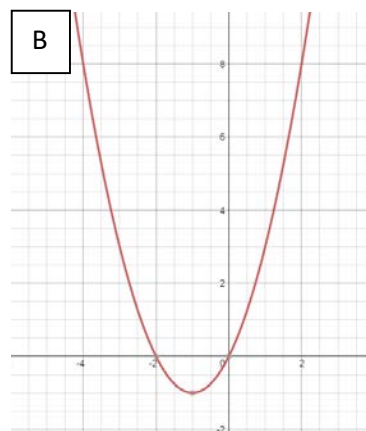
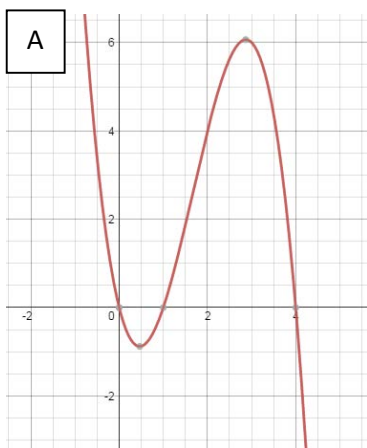
21) Graph the polynomial using your calculator to find the zeros and relative/local maximum and minimum. Sketch the graph using these points. Round all values to the nearest tenth.

$$f(x) = -(x - 4)(x - 6)(x - 2)$$



- Zeros: _____
- Relative/local Maximum: _____
- Relative/local Minimum: _____
- Degree: _____
- Leading Coefficient: _____
- End Behavior: as $x \rightarrow -\infty$, $f(x) \rightarrow$ _____
 as $x \rightarrow +\infty$, $f(x) \rightarrow$ _____

22) Select which graph is: $f(x) = x(x - 3)(x + 1)$



23) Factor each expression completely:

- a) $x^3 - 6x^2 + 5x + 12$ given that $(x - 4)$ is a factor
- b) $x^3 - 4x^2 + x + 6$ given that $(x - 3)$ is a factor

24) Perform the operations on complex numbers. Make sure to simplify your answer completely.

- a) $i =$
- b) $i^2 =$
- c) $(5 - 8i) + (1 + 3i)$
- d) $(2 + 3i) - (4 - 5i)$
- e) $(4 + 5i)(2 - 3i)$
- f) $(3 - i)(2 + i)$
- g) Solve for x : $-2x^2 - 50 = 0$

25) Divide using long or synthetic division:

- a) $(x^2 + 5x - 14) \div (x - 2)$
- b) $(x^2 - 2x - 48) \div (x + 5)$
- c) $(x^3 + x + 30) \div (x + 3)$
- d) $(8x^3 + 5x^2 - 12x + 10) \div (x - 2)$

26) Find the remainder of: $(2x^3 - 4x + 5) \div (x + 4)$

27) More factoring!

- a) $x^4 + 5x^2 - 36$
- b) $x^3 + 2x^2 - 5x - 10$
- c) $3x^2 + 10x + 8$
- d) $3x^5 - 12x^4$
- e) $3x^3 + x^2 - 12x - 4$

28) **Solve** each of the following for x . Round to 2 decimals if necessary.

- a) $3x^3 - 12x = 0$
- b) $5x^2 + 45 = 0$
- c) $x^4 - 81 = 0$
- d) $2x^2 - 3x - 9 = 0$
- e) $x^3 - 7x^2 - 2x + 14 = 0$
- f) $x^3 + 6x^2 - 9x - 54 = 0$

29) Write a function which would have the following characteristics:

- a) Crossing the x axis at $-2, 5$, and touching the x axis at 1 , with both ends of the graph moving towards positive infinity.

$$f(x) = \underline{\hspace{10cm}}$$

- b) Crossing the x axis at 0 and touching the x axis at 4 with the left end of the graph moving towards positive infinity and the right end of the graph moving towards negative infinity.

$$f(x) = \underline{\hspace{10cm}}$$

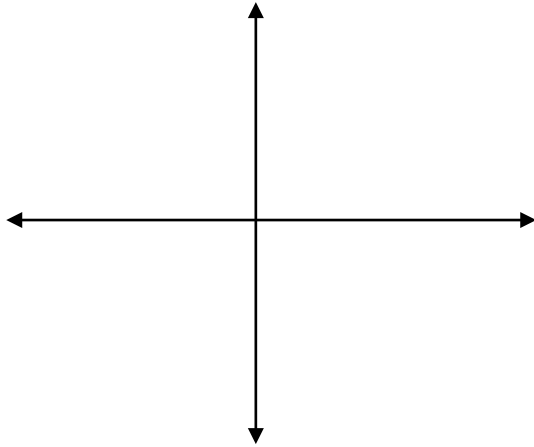
30) List all the possible rational roots of $2x^2 + 3x + 3 = 0$

31) Find all the zeros of the function $x^3 - 12x^2 + 35x - 24$

32) Is $(x + 5)$ a factor of $(x^3 + 4x^2 + 5x - 25)$? Show work to support your answer.

33) State the degree, the leading coefficient, and sketch the end behavior. Then sketch the graph by using that information and a table of values.

a) $y = x^3 + 3x^2 - 10x - 24$



-3	-2	-1	0	3

Identify the degree: _____

y-intercept: _____

Real solutions: _____

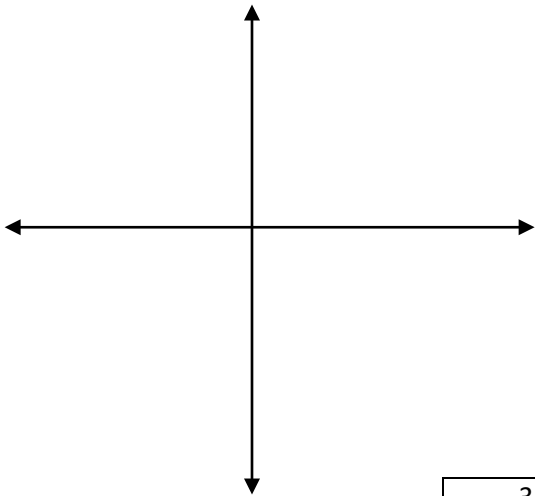
How many complex solutions does the equation have?

End behavior:

as $x \rightarrow \infty$ $f(x) \rightarrow$ _____

as $x \rightarrow -\infty$ $f(x) \rightarrow$ _____

b) $y = -x^4 - 3x^3 - 7x^2 - 15x - 10$



-3	-2	-1	0

Identify the degree: _____

y-intercept: _____

Real solutions: _____

How many complex solutions does the equation have?

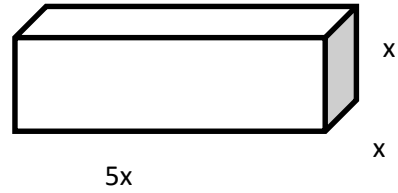
End behavior:

as $x \rightarrow \infty$ $f(x) \rightarrow$ _____

as $x \rightarrow -\infty$ $f(x) \rightarrow$ _____

34) Find the real and complex solutions of $g(x) = x^4 - 81$

35) The volume of the prism is 135 in^3 . Find the value of x using algebra. Show your equation and all work.



Exponents and Radicals:

- Skills:
 - Switch between radical and rational exponent form
 - Use properties of exponents and radicals to simplify expressions
 - Solve radical equations
 - Check for extraneous solutions
 - Know when solutions are \pm

36) Write in radical form:

- a) $x^{3/7}$
- b) $x^{1/3}$
- c) $(x^2)^{1/3}$

37) Write in exponential form:

- a) $\sqrt[3]{x^2}$
- b) $\sqrt{x^5}$
- c) $\sqrt[4]{x}$

38) Simplify each expression completely without using a calculator:

- | | |
|----------------------------|--|
| a) $x^5 \cdot x^2$ | f) $\left(\frac{x^2}{y^{-1}}\right)^{-1}$ |
| b) $\frac{x^8}{x^2}$ | g) $\sqrt[3]{\frac{64x^9}{8x^2}}$ |
| c) $(y^3)^4$ | h) $(a^2)^{\frac{3}{2}}$ |
| d) $5^{3/4} \cdot 5^{7/4}$ | i) $x^{\frac{1}{3}} \cdot x^{\frac{2}{5}}$ |
| e) $(3^{3/4})^{1/3}$ | |

39) Evaluate without using a calculator:

- | | |
|---------------------------------------|---------------------------------------|
| a) $(9)^{3/2}$ | e) $(\sqrt[3]{27})^2$ |
| b) $\frac{3^5}{3^2}$ | f) $\sqrt[3]{24}$ |
| c) $8^{-2/3}$ | g) $\left(\frac{1}{25}\right)^{-3/2}$ |
| d) $\left(\frac{1}{64}\right)^{-2/3}$ | h) 16^0 |

40) Solve algebraically and check for extraneous solutions:

a) $\sqrt{8x + 1} = 4 - x$

b) $\sqrt{x + 6} = x$

c) $2\sqrt[4]{x - 3} = 4$

d) $\sqrt[3]{2x + 4} + 6 = 10$

e) $x^3 = 8$

f) $x^3 = -27$

g) $x^4 = 81$

h) $x^4 - 2 = 254$

41) Show all work as you solve each radical equation algebraically or graphically.

a) $\sqrt{x + 25} = 4$

b) $\sqrt[3]{x} - 9 = -1$

c) $2\sqrt[3]{x - 3} = 4$

d) $x + 1 = \sqrt{7x + 15}$

42) The number of eggs, E , sold in a supermarket can be modeled by the equation $E = 4500\sqrt{0.5x + 2}$ where x is the number of days past since Monday. On how many days past Monday will 9000 eggs be sold?

Exponentials:

- Vocabulary
 - Principal/initial amount
 - Growth Rate
 - Compounded (quarterly, monthly, annually, continuously)
- Understanding Formulas:
 - General Exponential Form For Graphing: $y = a \cdot b^x$
 - Exponential Growth: $A = P(1 + r)^t$
 - Exponential Decay: $A = P(1 - r)^t$
 - Compounded Growth: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
 - Continuously Compounded Growth: $A = Pe^{rt}$

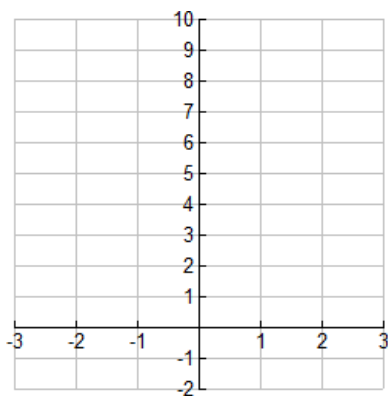
43) State the initial amount and the growth/decay rate (as a percent) for each model below.

a) $f(t) = 900e^{.12t}$

b) $f(t) = 560(1.04)^t$

c) $f(t) = 60(.85)^t$

- 44) You purchased a Honda Civic for \$22,912. You learned that your car will decrease in value by 15% each year. How much will your car be worth when you graduate from college in 5 years?
- 45) You deposit \$2500 in an account that earns 2.5% annual interest. Find the balance after one year if the interest is compounded monthly.
- 46) You want to have \$10,000 in your account after five years. Find the amount your initial deposit should be for each of the following situations.
- The account pays 3.5% interest compounded monthly.
 - The account pays 2.75% interest compounded quarterly.
 - The account pays 4.25% interest compounded continuously.
- 47) Determine the interest rate if an initial investment of \$3,500 became \$3,675 after one year.
- 48) How much money would you need to invest at 6% interest compounded annually for 8 years to end with \$9000?
- 49) Graph the function $f(x) = 3^x$ using the table.



x	y
-2	
-1	
0	
1	
2	

- 50) If your money grew from \$1500 to \$1842 in one year compounded annually, what was the interest rate (in percent form) rounded to the nearest hundredth?
- 51) The population of deer is currently 23,104 in Fairfield County and is growing at an annual rate of 3%. Write a model for the population of deer t years from now.
- 52) The number of books published in hard copy has been declining at an annual rate of 16%. Write a model for the number of books that will be published in hard copy t years from now if there are currently 35,944 published in hard copy.
- 53) The average weight W (in kilograms) of an Atlantic cod from the Gulf of Maine can be modeled by $W = 0.51(1.46)^x$ where x is the age in years.
- What is the initial weight of the cod?
 - What would the estimated weight of the cod be at age 6? Round to 2 decimal places.
 - When will the cod reach 15 kilograms? Round to 2 decimal places.