ALGEBRA TOPICS

FACTORING POLYNOMIALS AND RATIONALS: basic factoring and factoring of binomials, fractional exponents and binomials within rational expressions to simplify.

Factor completely:

1) $x^3(x+4)^5 - 2x^2(x+4)^6$

$x^2(x+4)^5[x-2(x+4)]$

$x^2(x+4)^5(-x-8)$

OR $-x^2(x+4)^5(x+8)$

2) $2x^{\left(\frac{1}{2}\right)}(x+2)^2 + 2x^{\left(\frac{3}{2}\right)}(x+2)^3$

$2x^{1/2}(x+2)^2[1+x(x+2)]$

$2x^{1/2}(x+2)^2(1+x^2+2x)$

$2x^{1/2}(x+2)^2(x+1)^2$

3) $\dfrac{8x(x+5)^3-4x^2(x+5)^2}{16x^2+80x}$

$\dfrac{\cancel{4x}(x+5)^{\cancel{2}\,1}[2(x+5)-x]}{4\,\cancel{16x}(\cancel{x+5})}$

$\dfrac{(x+5)(x+10)}{4}$ OR $\frac{1}{4}(x+5)(x+10)$

4) $\dfrac{3(2x^2-8)+12(x+2)^2}{12x^2+6x-36}$

$\dfrac{6(x^2-4)+12(x+2)^2}{6(2x^2+x-6)} = \dfrac{\cancel{6}(\cancel{x+2})[(x-2)+2(x+2)]}{\cancel{6}(2x-3)(\cancel{x+2})}$

$= \dfrac{3x+2}{2x-3}$

SOLVING POLYNOMIAL INEQUALITIES

Example 1: $x^2-4x-5<0$

Solution: $(x+1)(x-5)<0$ *factor and determine critical values: $x = -1, 5$*

+   -   +

-1   5

Answer: (-1, 5) *mark the zeros; pick a test point to determine the sign of the polynomial in each interval- this is called a SIGN CHART!*

PRACTICE PROBLEMS - Include a Sign Chart

5) $x^3+7x^2+10x > 0$

$x(x^2+7x+10)>0$

$x(x+5)(x+2)>0$

C.V. $x=0,-5,-2$ -5   -2   0   − + − +

Solution: $(-5,-2)\cup(0,\infty)$

6) $3x(x+2)^3+9x^2(x+2)^2 \le 0$

$3x(x+2)^2[(x+2)+3x] \le 0$

$3x(x+2)^2(4x+2)\le 0$ OR $6x(x+2)^2(2x+1)\le 0$

C.V. $x=0,-2,-1/2$ -2   -1/2   0   + + − +

Solution: $x=-2$ and $[-1/2, 0]$

PRE-CALCULUS TOPICS

Analyzing Graphs:

Domain and Range, x-intercepts (zeros) and y-intercepts, extrema (local and absolute)

End Behavior (limits at infinity): $\lim_{x\to\pm\infty} f(x)$

Continuity and Discontinuity: All Polynomials are continuous for all x.

Types of discontinuities: removable (graph as hole), removable with point (piece-wise), infinite (vertical asymptote) and jump (piece-wise.

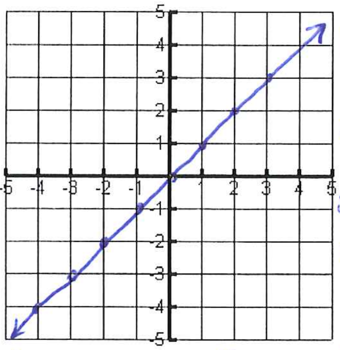
Even/Odd Functions:

Even: Symmetric to the y-axis. Algebraically: $f(-x) = f(x)$

Odd: Symmetric to the origin. Algebraically: $f(-x) = -f(x)$

Parent Functions that all graduating Pre-Calculus students should be able to sketch without the use of a calculator. Determine any key characteristics, domain, interval of continuity, (i.o.c.) symmetry, end behavior and type and location of any discontinuities.

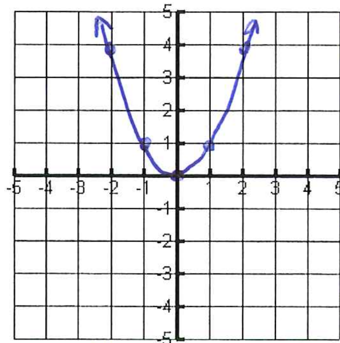
Linear: $y = x$ $m=1$ $b=1$



Domain: $(-\infty, \infty)$
i.o.c: $(-\infty, \infty)$
Symmetry: $f(-x) = -x$ odd
No discont.

E.B: $\lim_{x\to-\infty} y = -\infty$ $\lim_{x\to\infty} y = \infty$

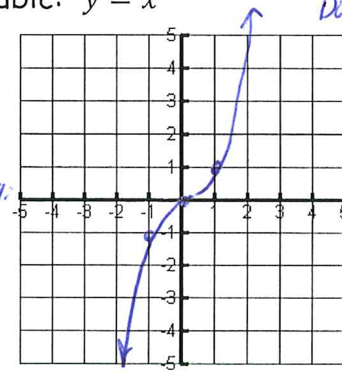
Quadratic: $y = x^2$ vertex: $(0,0)$



Domain: $(-\infty, \infty)$
i.o.c: $(-\infty, \infty)$
Symmetry: $f(-x) = x^2$ even
No discont.

E.B.: $\lim_{x\to-\infty} y = \infty$ $\lim_{x\to\infty} y = \infty$

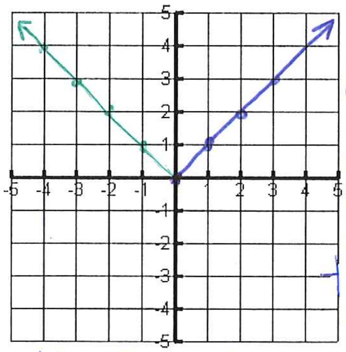
Cubic: $y = x^3$



Domain: $(-\infty, \infty)$
i.o.c. $(-\infty, \infty)$
Symmetry: $f(-x) = -x^3$ odd
No discont.

E.B. $\lim_{x\to-\infty} y = -\infty$ $\lim_{x\to\infty} y = \infty$

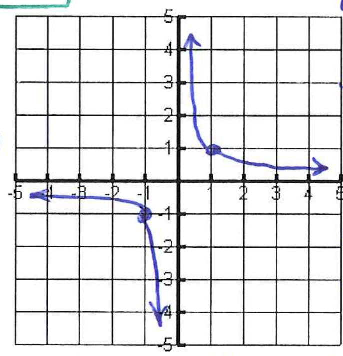
Absolute Value: $y = |x| = \begin{cases} x, if\ x \geq 0 \\ -x, if\ x < 0 \end{cases}$



vertex: $(0,0)$
Domain: $(-\infty, \infty)$
i.o.c: $(-\infty, \infty)$
Symmetry: $f(-x) = |-x| = |x|$ even.
no discont.

E.B. $\lim_{x\to-\infty} y = \infty$ $\lim_{x\to\infty} y = \infty$

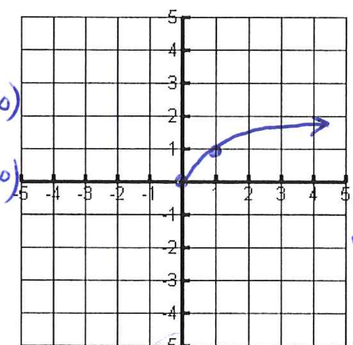
Rational: $y = \frac{1}{x}$



VA: $x=0$
HA: $y=0$
Domain: $(-\infty,0)\cup(0,\infty)$
i.o.c.: $(-\infty,0)\cup(0,\infty)$
$f(-x) = -\frac{1}{x}$
Symmetry: odd

E.B: $\lim_{x\to-\infty} y = \lim_{x\to\infty} y = 0$

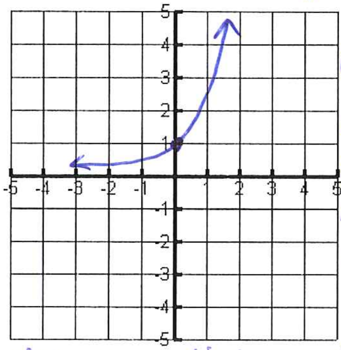
Root: $y = \sqrt{x}$



Domain: $[0, \infty)$
i.o.c. $(0,\infty)$ $x=0$ is cont. from the right
symmetry $f(-x) = \sqrt{-x}$ no symmetry
no point disc.

E.B. $\lim_{x\to-\infty} y = DNE$ $\lim_{x\to\infty} y = \infty$

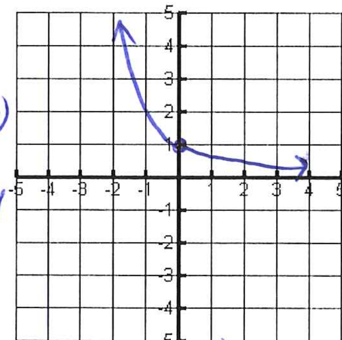
Exponential Growth: $y = e^x$ where $x > 0$ pt. $(0,1)$



HA: $y=0$
Domain: $(-\infty, \infty)$
i.o.c.: $(-\infty, \infty)$
$f(-x) = e^{-x}$
no symmetry
no discont.

E.B. $\lim_{x\to-\infty} y = 0$ $\lim_{x\to\infty} y = \infty$

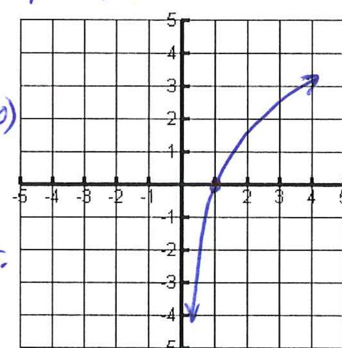
Exponential Decay: $y = e^x$ where $x < 0$ pt. $(0,1)$ HA: $y=0$



Domain: $(-\infty, \infty)$
i.o.c: $(-\infty, \infty)$
$f(-x) = e^{-x}$
no Symm.
no discont.

E.B. $\lim_{x\to-\infty} y = \infty$ $\lim_{x\to\infty} y = 0$

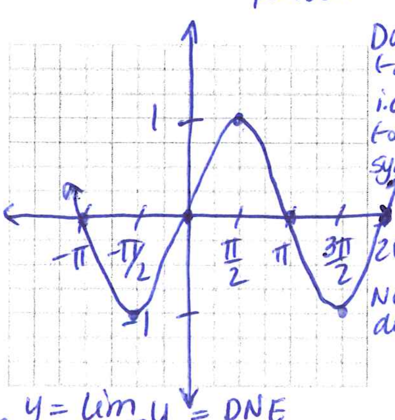
Logarithmic Growth: $y = \ln x$ pt. $(1,0)$ VA: $x=0$



Domain: $(0, \infty)$
i.o.c: $(0, \infty)$
Symmetry: $f(-x) = \ln(-x)$
no symm.
no pt. disiontinuit

E.B: $\lim_{x\to-\infty} y = DNE$ $\lim_{x\to\infty} y = \infty$

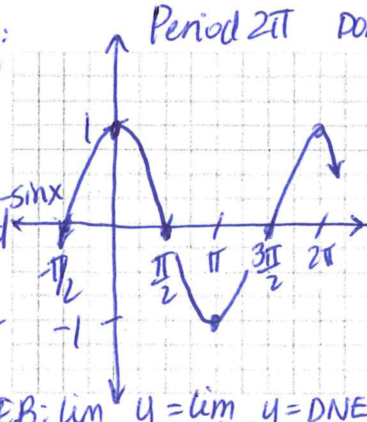
Sine: $y = \sin(x)$ period: 2π



Domain: $(-\infty, \infty)$
i.o.c $(-\infty, \infty)$
symm. $f(-x) = -\sin x$ odd
No discont

$\lim y = \lim y = DNE$

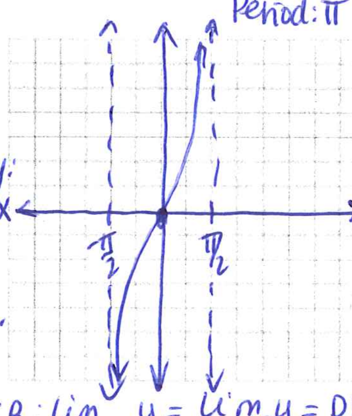
Cosine: $y = \cos(x)$ Period 2π



Domain: $(-\infty, \infty)$
i.o.c: $(-\infty, \infty)$
Symmetry: $f(-x) = \cos x$ even
no disiont.

E.B: $\lim y = \lim y = DNE$

Tangent: $y = \tan(x)$ Period: π



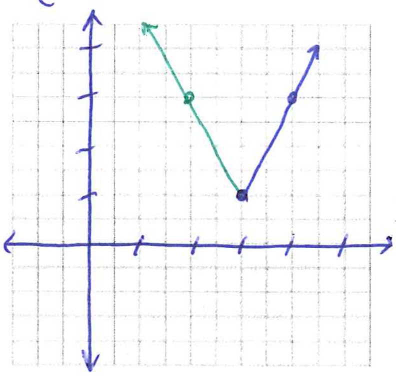
Domain: $x \neq \frac{\pi}{2} n$
i.o.c. $x \neq \frac{\pi}{2} n$
Symmetry: $f(-x) = -\tan x$ odd
Discontinuities $x = \frac{\pi}{2} n$ infinit

E.B: $\lim y = \lim y = DNE$

7 - 8 Write the absolute value function as a piecewise function then identify the vertex and sketch the graph.

7) $f(x) = 2|x-3| + 1$

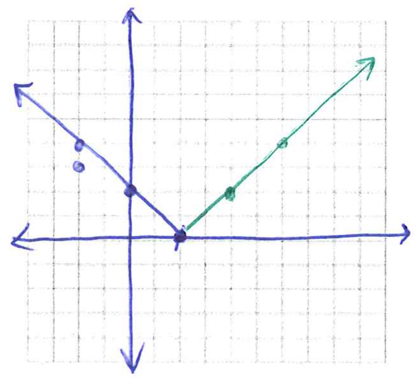
$$f(x) = \begin{cases} 2(x-3)+1, & x \geq 3 \\ -2(x-3)+1, & x < 3 \end{cases} = \begin{cases} 2x-5; & x \geq 3 \\ -2x+7, & x < 3 \end{cases}$$



vertex: $(3,1)$

8) $f(x) = |1-x|$

$$f(x) = \begin{cases} 1-x, & x \leq 1 \\ -(1-x), & x > 1 \end{cases} = \begin{cases} 1-x, & x \leq 1 \\ -1+x, & x > 1 \end{cases}$$



vertex: $(1,0)$

Analyze the following functions without graphing them on a graphing calculator (except where noted).

9) $f(x) = x^2 - 2x - 3 = (x-1)^2 - 1 - 3 = (x-1)^2 - 4$

$(x-3)(x+1)$

Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

x-intercept(s): $x = 3, -1$

y-intercept(s): $y = -3$

Extrema (by hand): absolute max $= -4$

End Behavior:

$\lim_{x\to -\infty} f(x) = \infty$

$\lim_{x\to \infty} f(x) = \infty$

Interval of Continuity:

$(-\infty, \infty)$

Tests for Symmetry:

$f(-x) = (-x)^2 - 2(-x) - 3$

$= x^2 + 2x - 3$ none

10) $g(x) = 6(x+2)^2 - 4(x+2)^3 = 2(x+2)^2[3 - 2(x+2)]$

$= 2(x+2)^2(-2x-1)$

$= -2(x+2)^2(2x+1)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x-intercept(s): $x = -2, -1/2$

y-intercept(s): $y = -8$

End Behavior: leading term: $(-2)(x^2)(2x) = -4x^3$ ↑ ↓

$\lim_{x\to -\infty} g(x) = \infty$

$\lim_{x\to \infty} g(x) = -\infty$

Interval of Continuity:

$(-\infty, \infty)$

Rational Expressions and Functions:

Domain: excludes all zeros of the denominator – both removable and non-removable.
Range: effected by any horizontal asymptotes. Recall, a function may cross a horizontal asymptote.
End Behavior (limits at infinity): $\lim_{x\to\pm\infty} f(x) = horizontal\ and\ Oblique\ asymptote$

Discontinuity: occur at the zeros of the denominator:

1. Zeros of the denominator that CANNOT be simplified: infinite discontinuities graph as vertical Asymptotes.
2. Zeros of the denominator that CAN be simplified: removable discontinuities graph as holes.

Simplifying: factor out and cancel common factors, simplify complex fractions by multiplying through by the LCD over itself.
Solving: relate terms to zero, combine all terms, determine the zeros of the numerator and denominator (called critical values) and create a sign chart to determine the signs before, between and after all critical values.

11) Determine the following for the function $f(x) = \frac{3x^2+2x-1}{2x^2+x-1} = \frac{(3x-1)(x+1)}{(2x-1)(x+1)}$ HA: $y = 3/2$

Domain: $(-\infty,-1)\cup(-1,1/2)\cup(1/2,\infty)$ Range: $(-\infty,3/2)\cup(3/2,\infty)$

x-intercept(s): $x = 1/3$ OR $(1/3, 0)$ y-intercept(s): $y=-2$ OR $(0,-2)$

End Behavior:

$\lim_{x\to-\infty} f(x) = 3/2$ $\lim_{x\to\infty} f(x) = 3/2$

Interval of Continuity: $(-\infty,-1)\cup(-1,1/2)\cup(1/2,\infty)$ Name and Location of Discontinuities:
$x=-1$ removable
$x=1/2$ infinite

12) Simplify

a. $\frac{3e^x x^2-3e^x}{e^{4x}(x+1)^3} = \frac{3e^x(x^2-1)}{(e^x)^{4}(x+1)^3}$

$\frac{3\cancel{(x+1)}(x-1)}{e^{3x}(x+1)^{\cancel{3}2}}$

$= \frac{3(x-1)}{e^{3x}(x+1)^2}$

b. $\frac{3x^{-2}}{1x^{-1}-\frac{1}{3}x^{-4}} = \frac{\left(\frac{3}{x^2}\right)}{\left(\frac{1}{x}-\frac{1}{3x^4}\right)} \cdot \frac{3x^4}{3x^4}$

$\frac{9x^2}{3x^3-1}$

c. $\frac{\frac{1}{x+1}}{\left(\frac{2}{x-3}-\frac{1}{x^2-2x-3}\right)} \cdot \frac{(x+1)(x-3)}{(x+1)(x-3)}$

$(x-3)(x+1)$

$\frac{x-3}{2(x+1)-1} = \frac{x-3}{2x+1}$

13) Solve

a. $\frac{x+3}{x-1} \le 4$

$\frac{x+3}{x-1} - 4 \le 0$

$\frac{x+3-4(x-1)}{x-1} \le 0$

$\frac{-3x+7}{x-1} \le 0$

C.V. $x = 7/3$ and 1

	1		7/3	
−	und	+	=0	−

$(-\infty,1)\cup[7/3,\infty)$

b. $\frac{5x(x+1)+10(x+1)^2}{20(x+2)(x+1)^3} \le 0$

$\frac{\cancel{5}(\cancel{x+1})[x+2(x+1)]}{4\,\cancel{20}(x+2)(x+1)^{\cancel{3}2}} \le 0$

$\frac{3x+2}{4(x+2)(x+1)^2} \le 0$

C.V. $x = -2/3, -2, -1$

	−2		−1		−2/3	
+	und	−	und	−	=0	+

$(-2,-1)\cup(-1,-2/3)$

FUNCTION OPERATIONS AND COMPOSITION; INVERSE FUNCTIONS

Composition of a function g with a function f is defined as:

$$h(\mathrm{x}) = g(f(x))$$

The domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.

Inverses: Functions f and g are inverses of each other provided:

$$f(g(x)) = x \text{ and } g(f(x)) = x$$

The function g is denoted as f^{-1}, read as "f inverse" and $f^{-1}(x) \neq \frac{1}{f(x)}$

Horizontal Line Test: If any horizontal line which is drawn through the graph of a function f intersects the graph no more than once, then f is said to be a **one-to-one** function and has an inverse.

14) Let f and g be functions whose values are given by the table below. Assume g is one-to-one.

x	$f(x)$	$g(x)$
1	6	2
2	9	3
3	10	4
4	-1	6

a. $f(g(3))$ $= f(4) = -1$

b. $g^{-1}(4)$ $g(x): (3,4) \leftrightarrow g^{-1}(x) = (4,3)$

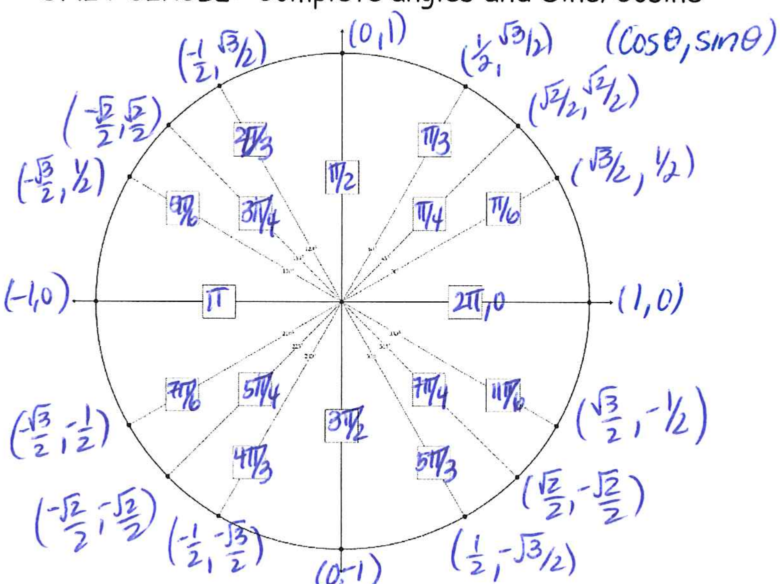
$\therefore g^{-1}(4) = 3$

c. $f(g^{-1}(6))$ $g^{-1}(6) = 4 \quad f(4) = -1$

d. $f^{-1}(f(g(2)))$ $= f^{-1}(f(3)) = f^{-1}(10) = 3$

TRIGONOMETRY

UNIT CIRCLE: Complete angles and Sine/Cosine



Key Trig Identities – formulas you should know:

$$\sin^2 x + \cos^2 x = 1$$
$$\sin(2x) = 2\sin x \cos x$$
$$\cos(2x) = 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$\sin(-x) = -\sin x \quad ODD$$
$$\cos(-x) = \cos x \quad EVEN$$

15) Without using a calculator or table, find each value:

a. $\cos\left(-\frac{\pi}{3}\right)$ $\frac{1}{2}$

b. $\sec\left(\frac{11\pi}{6}\right)$ $\frac{2\sqrt{3}}{3}$

c. $\tan^{-1}(-1)$ $-\frac{\pi}{4}$

d. $\sec^{-1}(-2)$ $\frac{2\pi}{3}$

16) Solve the trigonometric equations algebraically by using identities and **without** the use of a calculator. Find all solutions in the interval $0 \le \theta \le 2\pi$.

a. $2\sin^2\theta - 1 = 0$

$2\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

$\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

b. $\sin(2x) = \cos x$

$2\sin x\cos x = \cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\downarrow \qquad \searrow$

$x = \pi/2, 3\pi/2, \pi/6, 5\pi/6$

EXPONENTIAL AND LOGARITHMIC FUNCTIONS:

Logarithm: For any positive numbers b and y with $b \neq 1$, we define the **logarithm of y with base b** as follows: $\log_b y = x$ if and only if $b^x = y$

LAWS OF LOGARITHMS (for M, N, $b > 0,\ b \neq 1$)

$(i)\ \log_b MN = \log_b M + \log_b N$

$(ii)\ \log_b \frac{M}{N} = \log_b M - \log_b N$

$(iii)\ \log_b M = \log_b N \textit{ if and only if } M = N$

$(iv)\ \log_b M^k = k \cdot \log_b M$

$(v)\ b^{\log_b M} = M$

The logarithmic and exponential functions are **inverse** functions.

Example: Consider $f(x) = 2^x$ and $g(x) = \log_2 x$. Verify that (3,8) is on the graph of f and (8,3) on the graph of g. In addition, $f(g(x)) = 2^{\log_2 x} = x$ and $g(f(x)) = \log_2(2^x) = x$ which verifies that f and g are indeed inverse functions.

Simplify.

17) $\log_5 \frac{1}{125} = \log_5 5^{-3}$

$= -3$

18) $\log_4 \sqrt[8]{16}$

$\log_4 (4^2)^{1/8}$

$= \log_4 4^{2/8} = 1/4$

19) $2\ln\frac{3}{e^3} = \ln\left(\frac{3}{e^3}\right)^2 = \ln\left(\frac{9}{e^6}\right)$

$= \ln 9 - \ln e^6$

$= \ln(9) - 6$ OR $2\ln(3) - 6$

20) $\frac{1}{t}\ln e^t$

$\frac{1}{t}(t) = 1$

21) $e^{3\ln(x+5)}$

$e^{\ln(x+5)^3}$

$= (x+5)^3$

22) $e^{-\ln x}$

$= e^{\ln(x)^{-1}}$

$= x^{-1}$ OR $\frac{1}{x}$

23) $4\ln e^{x^2}$

$4x^2$

24) $e^{3x+\ln 5}$

$e^{3x} \cdot e^{\ln 5}$

$= 5e^{3x}$

25) $e^{3(\ln(x)+\ln 5)}$

$= e^{3(\ln(5x))}$

$= e^{\ln(5x)^3} = (5x)^3 = 125x^3$

26) Write $5\,ln\,(x) - 3\,ln\,(y) + 2\,ln\,(4x) - 6\,ln\,(y^2)$ as a a single logarithm.

$\ln x^5 - \ln y^3 + \ln 16x^2 - \ln y^{12} = \ln\left(\frac{x^5 \cdot 16x^2}{y^3 \cdot y^{12}}\right) = \ln\left(\frac{16x^7}{y^{15}}\right)$

27) Expand $\ln\left(\frac{3x^2}{2\sqrt{y^2-4}}\right)$.

$= \ln 3 + \ln x^2 - \ln 2 - \ln(y^2-4)^{1/2} = \ln 3 + 2\ln x - \ln 2 - \frac{1}{2}\ln(y^2-4)$

28) Evaluate:

a. $\ln 1$	b. $\ln 3e$	c. $\ln e$	d. $\ln 0$	e. $\ln(\ln e)$
0	$\ln 3 + \ln e$ $\ln(3) + 1$	1	DNE	$\ln(1) = 0$

Limits: $\lim_{x\to c} f(x) = N$ "Limit as x approaches c of f(x) equals N"

LIMIT ≠ CONTINUITY

Understanding the difference between limits and continuity: Limit is the value (y) that the function APPROACHES as you get close to c (either from one side or from both sides). Continuity implies that the limit exists (from both sides) and that the limit = the value at c!

Limit Exists: $\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) \;\therefore\; \lim_{x\to c} f(x)\ exists$ and will equal all the same value or $\pm\infty$.

Continuous: $\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)\ AND = f(c) = A\ (where\ A\ is\ a\ constant\ not\ \pm\infty)$ **MUST SHOW ALL 3!!**

One-Sided limit: exists if graph approaches a value from one side: + (right) or – (left)

$\lim_{x\to a} C = C$

Evaluating Limits:

1. **Tables:** either from table or by graphing and viewing table looking for values of y approaching the same number. Check one sided limits first and if they are equal the overall limit exists.
2. **Graphically:** Check one sided limits (again – value of limit is the y-value or the height of the graph) and if they are equal overall limit exists.
3. **Algebraically – Direct substitution:** plug in c. If you get a constant then limit exists. Rational functions may need to be simplified first!
   When you use Direct substitution and you get
   - $\frac{0}{number}$ then the limit is equal to 0
   - $\frac{0}{0}$ most likely a removable discontinuity, try simplifying or rationalizing.
   - $\frac{number}{0}$ most likely an infinite discontinuity, look at each side and evaluate one sided limit or if asking for two sided limit make sure the function is approaching the same from both sides.

$\lim_{x\to\pm\infty} f(x)$: **End Behavior:**

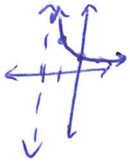
- Polynomial: use degree (even or odd) and sign of leading coefficient to decide parabolic or cubic.
- Rational: end behavior is determined by the Horizontal or Oblique Asymptotes.
- Other "Known" Graphs: know the graphs!

Examples: Evaluate the limits:

29) $\lim_{x\to 3}(2x^2)$

$2(3)^2=2(9)=18$

30) $\lim_{x\to -2^+}\frac{1}{x+2}$ $=\frac{1}{0}=\frac{\#}{0}$ VA!



x	y
-1	1
0	$1/2$
1	$1/3$

$\lim_{x\to -2^+}\frac{1}{x+2}=\infty$

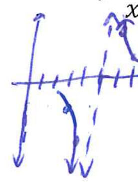
31) $\lim_{x\to \frac{\pi}{2}}(x\sin x)$

$=\frac{\pi}{2}\sin(\frac{\pi}{2})$

$=\frac{\pi}{2}(1)=\frac{\pi}{2}$

32) $\lim_{x\to 5}\frac{3}{x-5}$ $=\frac{3}{0}=\frac{\#}{0}$ VA

both sides



x	y
3	$-3/2$
4	-3
6	3
7	$3/2$

$\lim_{x\to 5^-}\frac{3}{x-5}=-\infty$

$\lim_{x\to 5^+}\frac{3}{x-5}=\infty$

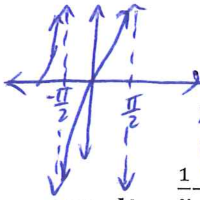
$\lim_{x\to 5}\frac{3}{x-5}$ = D N E

33) $\lim_{x\to 0^+}\ln(x)$ $=-\infty$

"Known" graph

34) $\lim_{x\to -\frac{\pi}{2}}\tan(x)$

"known" graph



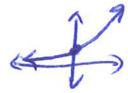
$=$ DNE b/c

$\lim_{x\to -\pi/2^-}\tan(x)=\infty$

$\lim_{x\to -\pi/2^+}\tan x=-\infty$

35) $\lim_{x\to -\infty}3e^{2x}$

E.B. "known" graph



$\therefore =0$

36) $\lim_{x\to 1}\frac{3x^2-2x-1}{x^2-1}$

$=\frac{3-2-1}{1-1}=\frac{0}{0}$ removable

$=\lim_{x\to 1}\frac{(3x+1)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

$=\frac{3+1}{1+1}=\frac{4}{2}=2$

37) $\lim_{x\to\infty}\frac{2x}{x-1}$

E.B. HA: $y=\frac{2}{1}=2$

$\therefore \lim_{x\to\infty}\frac{2x}{x-1}=2$

38) $\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}$

$=\frac{1-1}{1-1}=\frac{0}{0}$ remov. E.B.

$\lim_{x\to 1}\frac{(\frac{1}{x}-1)}{(x-1)}\frac{x}{x}=\lim_{x\to 1}\frac{1-x}{(x-1)(x)}$

$=\lim_{x\to 1}\frac{-\cancel{(x-1)}}{\cancel{(x-1)}x}=\frac{-1}{1}=-1$

39) $\lim_{x\to -\infty}2x(x-3)^2(x+1)$

leading term: $2x(x^2)x$
$=2x^4$ ↑↑

$\lim_{x\to -\infty}2x(x-3)^2(x+1)=\infty$

40) $\lim_{x\to -\infty}\frac{3}{x-5}$

E.B. HA: $y=0$

$\lim_{x\to -\infty}\frac{3}{x-5}=0$

41) If $f(x)=\begin{cases}2x-3, & if\ x<-1\\ 2, & if\ x=-1\\ 5x, & if\ x>-1\end{cases}$, determine the following:

a. $\lim_{x\to -1^-}f(x)$

$\to x<-1$

$=2(-1)-3=-2-3=-5$

b. $\lim_{x\to -1^+}f(x)$

$\to x>-1$

$=5(-1)=-5$

c. $\lim_{x\to -1}f(x)=-5$

b/c $\lim_{x\to -1^-}f(x)=-5$ (=)

$\lim_{x\to -1^+}f(x)=-5$

d. Is f continuous?

No, b/c

$\lim_{x\to -1}f(x)=-5\neq f(-1)=2$

42) If $f(x)=|x+2|=\begin{cases}x+2, & x\geq -2\\ -(x+2), & x<-2\end{cases}$, determine the following:

a. $\lim_{x\to -2^-}f(x)$

$\to x<-2$

$\lim_{x\to -2^-}f(x)=-2+2=0$

b. $\lim_{x\to -2^+}f(x)$

$x>-2$

$\lim_{x\to -2^+}f(x)=-(-2+2)=0$

c. $\lim_{x\to -2}f(x)=0$

b/c

$\lim_{x\to -2^-}f(x)=0$

$\lim_{x\to -2^+}f(x)=0$

d. Is f continuous?

Yes, b/c

$\lim_{x\to -2}f(x)=f(-2)=0$

43) Determine whether $f(x)$ is continuous. Justify. Then, determine domain, interval of continuity and location and type of any discontinuities.

a. $f(x)=\begin{cases}3x^2, & x<0\\ 4, & x=0\\ 3\sin^2 x, & x>0\end{cases}$

$\lim_{x\to 0^-}f(x)=3(0)^2=0$ $\therefore \lim_{x\to 0}f(x)=0$

$\lim_{x\to 0^+}f(x)=3(\sin 0)^2=0$

$f(0)=4$ $\therefore$ discontinuous at $x=0$ removable w/ pt

b. $f(x)=\begin{cases}\ln x, & x<e^2\\ \sqrt{x}, & x\geq e^2\end{cases}$

$\lim_{x\to e^{2-}}f(x)=\ln e^2=2$

$\lim_{x\to e^{2+}}f(x)=f(e^2)=\sqrt{e^2}=e$

$2\neq e$ $\therefore$ discont. at $x=e^2$

jump.

c. $f(x)=\begin{cases}3x^2-1, & x\leq 2\\ 5x+1, & x>2\end{cases}$

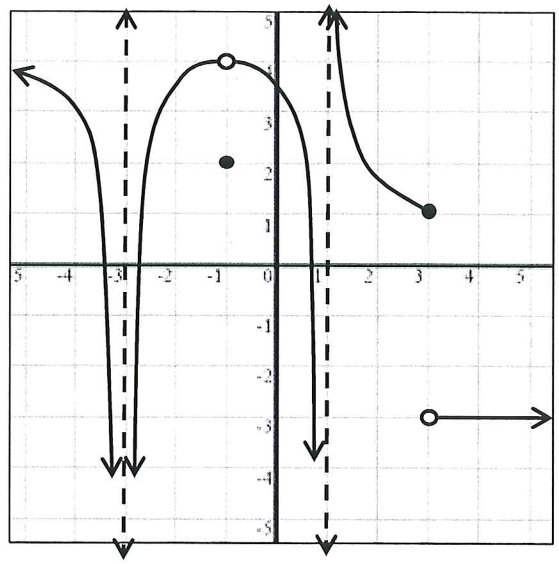
$\lim_{x\to 2^-}f(x)=f(2)=3(2)^2-1=12-1=11$

$\lim_{x\to 2^+}f(x)=5(2)+1=11$

$\therefore \lim_{x\to 2}f(x)=f(2)=11$

Cont.!

44) Let f(x) be the graph below. Determine the following:



a. $\lim_{x\to-3^-} f(x)$ = $-\infty$

b. $\lim_{x\to-3^+} f(x)$ = $-\infty$

c. $\lim_{x\to-3} f(x)$ = $-\infty$

d. f(-3) = und.

e. Is f continuous at -3? NO, infinite discontinuity

f. $\lim_{x\to-1^-} f(x)$ = 4

g. $\lim_{x\to-1^+} f(x)$ = 4

h. $\lim_{x\to-1} f(x)$ = 4

i. f(-1) = 2

j. $\lim_{x\to1^-} f(x)$ = $-\infty$

k. $\lim_{x\to1^+} f(x)$ = ∞

l. $\lim_{x\to1} f(x)$ = DNE b/c $-\infty \neq \infty$

m. $f(1)$ = und.

n. $\lim_{x\to3^-} f(x)$ = 1

o. $\lim_{x\to3^+} f(x)$ = -3

p. $\lim_{x\to3} f(x)$ = DNE b/c $1 \neq -3$

q. $f(3)$ = 1

r. $\lim_{x\to-\infty} f(x)$ = ∞

s. $\lim_{x\to\infty} f(x)$ = -3

t. Continuity Interval: $(-\infty,-3)\cup(-3,-1)\cup(-1,1)\cup(1,3)\cup(3,\infty)$

u. Location and type of each discontinuity:
x = -3 infinite
x = -1 removable pt
x = 1 infinite
x = 3 jump

v. Domain: $(-\infty,-3)\cup(-3,1)\cup(1,\infty)$

45) Draw a graph given the following conditions:

- $f(0) = 0$
- $f(1) = 2$
- $f(4) = -3$

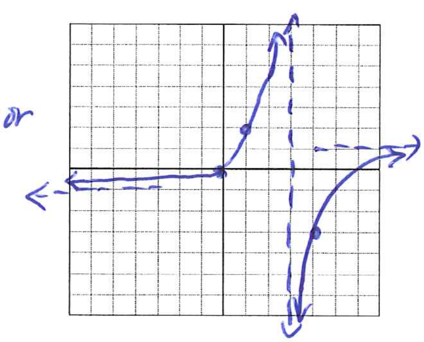
} pts.

- $\lim_{x\to-\infty} f(x) = -1$
- $\lim_{x\to\infty} f(x) = 1$

} End behavior

- $\lim_{x\to3^+} f(x) = -\infty$
- $\lim_{x\to3^-} f(x) = \infty$

VA at x = 3



46) Draw a graph given the following conditions:

- $f(-2) = 0$
- $f(2) = 2$

} pt.

- $\lim_{x\to-\infty} f(x) = -\infty$
- $\lim_{x\to\infty} f(x) = 5$

} End behavior

- $\lim_{x\to-2^-} f(x) = 4$ ← jump → $\lim_{x\to-2^+} f(x) = 0$