Algebra 12: Final Review 2018-2019

FORMULA SHEET

Slope: \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

Slope-Intercept Form: \[ y = mx + b \]

Point-Slope Form: \[(y - y_1) = m(x - x_1)\]

Standard Form: \[ Ax + By = C \]

Quadratic Formula: \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Standard Form of a Quadratic: \[ y = ax^2 + bx + c \]

Vertex Form of a Quadratic: \[ y = a(x - h)^2 + k \]
Part 1

Evaluate: (Section 1.2)

1. \( f(x) = -x - 3 \), for \( x = -3 \)
2. \( y = x^2 + 3 \), for \( x = -2 \)
3. \( f(x) = -x^2 + 5x - 8 \), for \( x = 4 \)

Solve each equation: (Sections 2.2, 2.3)

4. \(-x + 8 = -12\)
5. \(\frac{x}{-5} = -12\)

6. \(9x - (-7x) = -32\)
7. \(3(x - 10) = -36\)

8. \(\frac{x}{8} = \frac{4}{5}\)
9. \(\frac{x + 2}{5} = \frac{2x - 11}{7}\)

10. \(3(4 + 4x) = 12x + 12\)
11. \(9(x - 4) - 7x = 5(3x - 2)\)

Solve for the indicated variable: (Section 2.5)

12. \(20x - 10y = 5\) (solve for \(y\))
13. \(12x + 7y = 7\) (solve for \(y\))

14. \(y = 2x - 5\) (solve for \(x\))
15. \(3x - y = -2y\) (solve for \(y\))
Part 2

1. Solve each inequality. Represent your solutions on a number line: (Sections 3.2, 3.3, 3.4)

   a. \( 3 \leq -x + 2 \leq 8 \)

   b. \( 2(5x + 3) > 4x + 1 - 7 \)

   c. \( 42w \geq 2(w + 7) \)

   d. \( -2 \leq d + 1 < 4 \)

   e. \( 2t \leq -4 \) or \( 7t \geq 49 \)

   f. \( 5m < -10 \) or \( 3m > 9 \)
Write an inequality to describe the solutions shown on the number line: (Sections 3.1, 3.6)

2. 

3. 

Graph the solution to the inequality: (Section 6.5)

4. \(-y \leq \frac{1}{3}x + 2\)

5. \(x > 3\)

6. \(y \geq x + 4\)

7. \(y < 2x - 1\)
8. Solve each system of linear inequalities by graphing.

a. \[ y < 10x \]
   \[ y > x - 5 \]

b. \[ 2x - y > 0 \]
   \[ 3x + 2y \leq -14 \]

Write an inequality to describe the solution shown in the graph: (Section 6.5)

9.

10.

Graph the solution to the system below: (Section 6.6)

11. \[ \begin{cases} 2x - 6y \geq -18 \\ y > x^2 + 2x - 3 \end{cases} \]
Part 3

Find the slope of the line going through the two points:  (Section 5.1)

1. (-6, 2) and (4, 7)  
2. (-8, 5) and (-3, 5)  
3. (1, 0) and (1, -4)

Write an equation of the line (in slope-intercept form) with the following criteria:  (Section 5.3)

4. slope of \( \frac{2}{3} \) and y-intercept of -3  
5. slope of -4 and y-intercept of 10.  
6. slope of 5 and passing through (-2, 5). 
7. slope of \( \frac{-1}{3} \) and passing through (6, 6).  
8. going through points (5, 4) and (7, 8).
9. going through points (3, 7) and (3, -1).
10. horizontal line through point (6, 2). 
11. vertical line through point (-3, -5).
12. Write an equation for the line in the graph below: 
(Section 5.3)

13. Sketch the line \( y = -\frac{2}{3}x - 1 \)

14. Examine the graph below. Explain what real-world quantities the slope and \( y \)-intercepts represent. Then find the slope and \( y \)-intercept. (Section 5.3)

a) What is the slope? _________

b) Complete the sentence:

Each minute, the weight of a bag of popcorn _____________________________.

c) What is the \( y \)-intercept? __________

d) What does the \( y \)-intercept represent? Explain.

Find the \( x \)-intercept and the \( y \)-intercept of the line: (Section 5.5)

15. \(-6x + 12y = 18\) 

16. \(x - 3y = -9\)
Part 4
(all problems are sections 6.1-6.4)

1. List the **three** different methods to solve a system of equations.

2. Solve the system of equations. \[ \begin{align*}
    y &= 3x + 2 \\
    6x - 2y &= 8 
\end{align*} \]

   a. Describe what happened when you tried to solve the system.

   b. Graph the system of equations. How does the graph of the system explain what happened with the equations?

3. Solve the system of equations. \[ \begin{align*}
    18x - 3y &= 9 \\
    y &= 6x - 3 
\end{align*} \]

   a. Describe what happened when you tried to solve the system.

   b. Graph the system of equations. How does the graph of the system explain what happened with the equations?
4. Solve the following systems using the method of your choice. Check your solutions.

a) \[ \begin{align*}
    y &= 3x + 7 \\
    y &= -4x + 21
\end{align*} \]

b) \[ \begin{align*}
    3x - y &= 17 \\
    -x + y &= -7
\end{align*} \]

c) \[ \begin{align*}
    x &= 3y - 5 \\
    2x + 12y &= -4
\end{align*} \]

d) \[ \begin{align*}
    y &= 2x - 3 \\
    -y &= 2x - 1
\end{align*} \]

e) \[ \begin{align*}
    y &= -7 + 5x \\
    4x + 8y &= -12
\end{align*} \]

f) \[ \begin{align*}
    21x + 28y &= 14 \\
    9x + 12y &= 6
\end{align*} \]

g) \[ \begin{align*}
    2x - 3y &= 12 \\
    -x - 3y &= -6
\end{align*} \]

h) \[ \begin{align*}
    2x - 3y &= 1 \\
    -2x + 3y &= 1
\end{align*} \]
5. Bob climbed down a ladder from his roof, while Roy climbed up another ladder next to him. Each ladder had 30 rungs. Their friend Jill recorded the following information about Bob and Roy:

Bob went down 2 rungs every second. Roy went up 1 rung every second.

At some point, Bob and Roy were at the same height. Which rung were they on?

6. Is the ordered pair (1, -1) a solution to the following system of equations? Explain. Show your work!

\[ y = 3x - 4 \]
\[ y = -\frac{1}{2}x + 3 \]

7. Earl solved a system of equations using substitution. Did he do it correctly? How do you know? If he did not, find his error.

**System:**
\[
\begin{align*}
  x + y &= 3 \\
  2y - 3x &= 16 \\
\end{align*}
\]

**Earl’s solution:**
\[
\begin{align*}
  2(3 - x) - 3x &= 16 \\
  6 - x - 3x &= 16 \\
  6 - 4x &= 16 \\
  6 - 6 - 4x &= 16 - 6 \\
  -4x &= 10 \\
  -\frac{4x}{4} &= \frac{10}{4} \\
  x &= -2.5 \\
  y &= 5.5 \\
\end{align*}
\]
8. Adrian is in Algebra. He solved the system: \( y = 5x - 2 \) and \(-3x + 6y = -12\) and got the solution \((2, 8)\), but he’s not feeling very confident. Decide whether or not he is correct and convince him of your position.

9. As treasurer of his school’s FFA club, Kenny wants to buy gifts for all 18 members. He can buy t-shirts for $9 and sweatshirts for $15. The club has only $180 to spend. If Kenny wants to spend all of the club’s money, how many of each type of gift can he buy?

   a. Write a system of equations representing this problem.

   b. Solve your system of equations and figure out how many of each type of gift Kenny should buy.

10. The school that Stefan goes to is selling tickets to a choral performance. On the first day of ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of $38. The school took in $52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.

11. Matt and Mary are selling fruit for a school fundraiser. Customers can buy small boxes of oranges and large boxes of oranges. Matt sold 3 small boxes of oranges and 14 large boxes of oranges for a total of $203. The cost of the large box is $6 more than the cost of a small box. Find the cost of a large box and the cost of a small box.
Part 5

1. **What is the greatest common factor of the expression? (Section 8.2)**
   a) \(-16x^2 + 8x - 4\)  
   b) \(18x^3 + 12x^2 - 27x\)

2. **Add or subtract the polynomials (Write answers in standard form). (Section 8.1)**
   a. \((2x^2 + 120x) + (-5x^2 - 80)\)  
   b. \((x^3 - 3x^2 + 5x) - (6x^3 + 5x^2 + 12)\)

3. **Complete the table. (Section 8.1)**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Degree</th>
<th>Classify by Degree</th>
<th>Number of Terms</th>
<th>Classify by Number of terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2x^3 + 5x^2 - 7x + 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3x + 8x^4 + 7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2x - 3 + 8x^2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3x + 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8x^2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4m^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. **Multiply the expressions and simplify (Section 8.3)**
   a. \((6x - 11)(2x + 5)\)  
   b. \((-5x + 3)(x + 2)\)  
   c. \((12x + 1)(x - 2)\)

   d. \((-2x)(5x - 3)\)  
   e. \(4y(y^2 - 2y + 3)\)
5. Factor each expression completely. \((Sections 8.5, 8.6)\)

a) \(x^2 - x - 42\)  
b) \(3x^2 + 19x + 20\)  
c) \(x^2 - 14x + 33\)  
d) \(9x^2 - 100\)  
e) \(4x^2 - 4x + 1\)  
f) \(-3x^2 - 15x - 18\)  
g) \(2x^2 + 6x - 36\)  
h) \(2x^3 + 12x^2 - 5x - 30\)  
i) \(2x^3 + 6x^2 + 3x + 9\)
1. Complete the following table for the expression \( y = x^2 - 3x + 7 \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. **Describe the graph by answering the following questions.** *(Section 9.1)*
   
   a) What type of function does the graph represent? ________________
   
   b) What is the graph of this function called? ________________
   
   c) Where is the vertex? ________________
   
   d) Does the graph have a maximum or minimum? ________________
   
   e) Does the graph x-intercepts? If so, where? ________________
   
   f) Does the graph have a y-intercept? If so, where? ________________
   
   g) Draw the line of symmetry on the graph. Write the equation for the axis of symmetry. ________________
   
   h) What is the domain? ________________
   
   i) What is the range? ________________

3. **Graph the quadratic equation:** *(Sections 9.2-9.4)*
   
   a) \( y = x^2 - 6x + 4 \)  
   b) \( y = 2(x + 2)^2 - 3 \)  
   c) \( y = -(x - 2)(x + 6) \)
Graph and answer the questions for the problem below: (section 9.2)

4. A punter kicked the football into the air with an upward velocity of 62 ft/s. Its height $h$ in feet after $t$ seconds is given by the function $h = -16t^2 + 62t + 2$.

   ![Graph of height vs. time]

   a. What is the maximum height the ball reaches?

   b. How long will it take the football to reach the maximum height?

   c. How long does it take for the ball to hit the ground?

5. What are the x-intercepts of the graph to the right? (Section 9.3)

   ![Graph of quadratic function]

6. What is the y-intercept of the parabola? $y = 5x^2 + 2x - 3$

7. Label each graph as linear, quadratic, or exponential. (Section 9.7)

   ![Graphs of various functions]
8. Find the value of \( c \) that will make the expression a perfect square trinomial. \((Section 9.5)\)
\[ x^2 - 30x + c \]

9. Write the function in vertex form by completing the square. Then, state the vertex. \((Section 9.5)\)
   a) \( y = x^2 + 5x - 4 \)  
   b) \( y = x^2 + 2x - 28 \)

10. What methods do we know for solving quadratic equations? \((Chapter 9)\)

11. Solve each equation for \( x \). Use a method of your choice. \((Sections 9.4-9.6)\)
   a) \(-2x^2 + 3x + 10 = 0\)  
   b) \((2x - 1)(5x + 2) = 0\)  
   c) \(2x^2 - 128 = 0\)  
   
   d) \(x^2 + 11x - 26 = 0\)  
   e) \(x^2 - 25 = 0\)  
   f) \(x^2 - 19x + 80 = -8\)  
   
   g) \(5x^2 - 8x = 8 - 5x\)  
   h) \(-3x^2 + 7x = -10\)  
   i) \((x + 3)^2 = 25\)
12. How many real-number solutions does each equation have?

a) \( x^2 + 4x + 5 \)  
b) \( 3x^2 - 9x + 5 = 0 \)

13. You are creating a rectangular banner for a school pep rally. You have 100 \( \text{ft}^2 \) of paper, and you want the length to be 15 ft longer than the width. What should be the dimensions of the banner?

14. You throw a ball upward. Its height \( h \), in feet, after \( t \) seconds can be modeled by the function \( h = -16t^2 + 30t + 6 \). After how many seconds will it hit the ground?

15. Refer to the graph below of the following system:

\[
\begin{align*}
\text{a) } y &= x^2 - x - 2 \\
\text{b) } y &= -x + 2
\end{align*}
\]

a. Match each equation with its graph on the coordinate plane.

b. Identify the solutions: ( , ) and ( , )

c. \( y = -x+2 \) is greater than \( y = x^2 - x - 2 \) when \( x \) is between _____ and _____

16. Solve each system.

a) \[
\begin{align*}
y &= x^2 + 3x - 23 \\
y &= 25 - 5x
\end{align*}
\]

b) \[
\begin{align*}
y &= x^2 + 2x - 2 \\
y &= x + 10
\end{align*}
\]