

Name:
Period:

Review for Final 2019
Geometry 22

Note to student: This packet should be used as practice for the Geometry 22 final exam. This should not be the only tool that you use to prepare yourself for the exam. You must go through your notes, re-do homework problems, class work problems, formative assessment problems, and questions from your tests and quizzes throughout the year thus far.

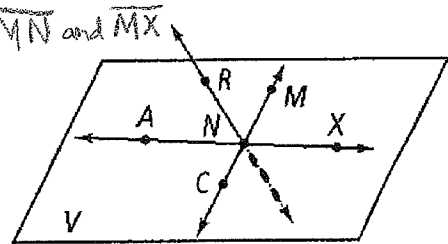
Section 1

1) Classify each statement as true or false, and explain your reasoning in each false case.

- a) Two planes intersect in only one point. False - they intersect in a line, which has infinitely many points.
- b) A ray starts at one point on a line and goes on forever. True
- c) The intersection of 2 planes is one line True
- d) Any four points are collinear. False - any two points are collinear

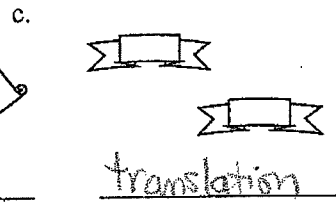
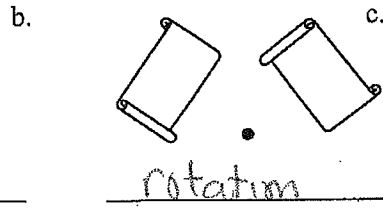
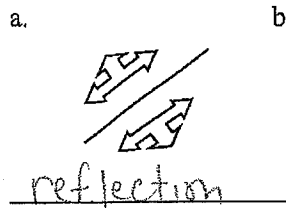
2) Use the figure below for #6-14. Note that \overline{RN} pierces the plane at N. It is not coplanar with V.

- a) Name two segments shown in the figure. Sample answer: \overline{MN} and \overline{MX}
- b) What is the intersection of \overline{CM} and \overline{RN} ? Point N
- c) Name three collinear points. Sample answer: Points A, N, and X
- d) What are two other ways to name plane V? Sample answer: $\square ANC$ and $\square MNX$
- e) Are points R, N, M, and X coplanar? no
- f) Name two rays shown in the figure. Sample answer: \overrightarrow{AX} and \overrightarrow{NC}
- g) Name ~~the~~ pair of opposite rays with endpoint N. Sample answer: \overrightarrow{NX} and \overrightarrow{NA}
- h) \overrightarrow{AN} is the same as \overrightarrow{NA} . True or False? True. They are the same line. (But, \overrightarrow{AN} and \overrightarrow{NA} are NOT the same ray.)
- i) \overline{ANX} names a plane. True or False?



False. You can't use 3 collinear points when naming a plane.

3) Below each figure write the name of the kind of rigid transformation shown.



Section 2

Complete the following statements:

- 1) $\angle ABC$ and $\angle BCD$ are complementary. $m\angle ABC = 6x^\circ$ and $m\angle BCD = 12x^\circ$. Find x .

$$6x + 12x = 90$$

$$18x = 90$$

$$x = 5$$

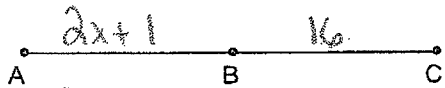
- 2) $\angle ABC$ and $\angle BCD$ are supplementary. $m\angle ABC = 40x^\circ$ and $m\angle BCD = 20^\circ$. Find x .

$$40x + 20 = 180$$

$$40x = 160$$

$$x = 4$$

- 3) $AB = 2x + 1$, $BC = 16$ inches, $AC = 5x - 4$. Use the diagram to solve for x :



$$2x + 1 + 16 = 5x - 4$$

$$2x + 17 = 5x - 4$$

$$17 = 3x - 4$$

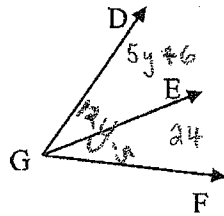
$$21 = 3x$$

$$x = 7$$

- 5) \overline{WS} bisects $\angle BWV$. $m\angle BWS = 32^\circ$. What is $m\angle BWV$?

$$64^\circ$$

- 4) Solve for y : $m\angle DGF = 12y - 5$, $m\angle EGF = 24^\circ$, $m\angle DGE = 5y + 6$



$$5y + 6 + 24 = 12y - 5$$

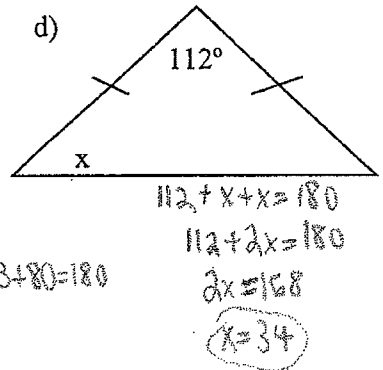
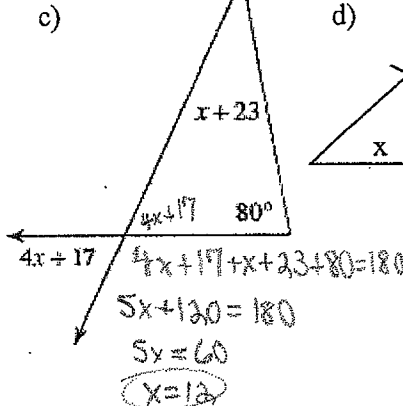
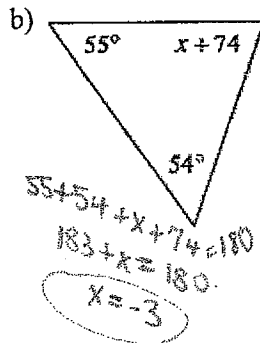
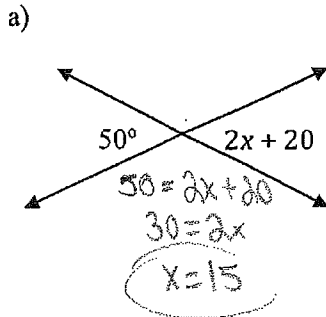
$$5y + 30 = 12y - 5$$

$$30 = 7y - 5$$

$$35 = 7y$$

$$y = 5$$

- 6) Determine the value of x :



- 7) Use the following steps to determine whether the given statement is a definition.

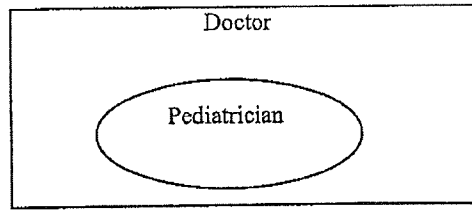
Linear pairs are supplementary, adjacent angles.

- a) Conditional statement If something is a linear pair, then it is a figure with supplementary and adjacent angles.
- b) Converse - If 2 angles are supplementary and adjacent, then they are a linear pair.
- c) Biconditional statement Something is a linear pair if and only if it is a figure with supplementary adjacent angles.
- d) Decide whether the statement is a definition. Explain your reasoning.

yes, because both the conditional and its converse are true.

8) Write the conditional statement that corresponds to the Venn diagram below:

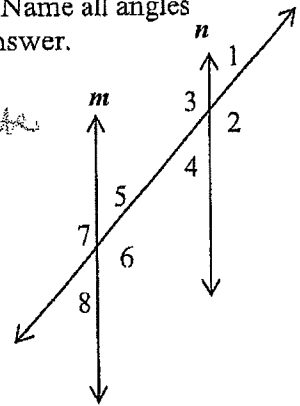
If someone is a pediatrician,
then they are a doctor.



Section 3

1) For the following exercises, refer to the diagram below. Lines m and n are parallel. Name all angles congruent to the given angle and give the theorems or postulates that justify your answer.

- a) $\angle 6 \cong \angle 7$, vertical angles thm; $\angle 3$, alt int \angle 's thm; $\angle 2$, corresp. \angle 's postulate
- b) $\angle 8 \cong \angle 5$ vert \angle thm; $\angle 1$, alt ext \angle thm; $\angle 4$, corr \angle post
- c) $\angle 5 \cong \angle 4$, alt int \angle thm; $\angle 1$ corr \angle 's post; $\angle 8$ vert \angle thm
- d) $\angle 7 \cong \angle 6$, vert \angle thm; $\angle 3$ corr \angle post; $\angle 2$ alt ext \angle thm



2) For the figure to the right $m\angle BCD = 160 - 3x^\circ$, and

$m\angle CFH = 35^\circ$. What is x ?

What Theorem or Postulate supports your answer?

$$160 - 3x + 35 = 180$$

$$195 - 3x = 180$$

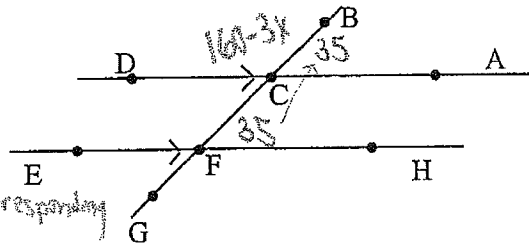
$$-3x = -15$$

$$x = 5$$

sample answer:

$\angle BCA$ and $\angle CFH$ are corresponding and therefore congruent.

$\angle DCB$ and $\angle BCA$ form a linear pair and are therefore supplementary.



3) For the figure to the right $m\angle BCA = 68^\circ$, and $m\angle CFH = 92 - 8x^\circ$. What value of x makes $\overline{AD} \parallel \overline{EH}$?

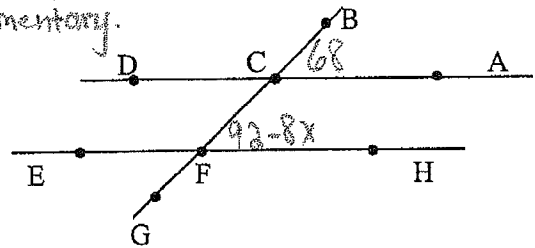
What Theorem or Postulate supports your answer?

$$68 = 92 - 8x$$

$$-24 = -8x$$

$$x = 3$$

$\angle BCA$ and $\angle CFH$ are corresponding angles and therefore congruent.



4) Fill in the blanks so that the sentences are true.

a) The sum of angles in any quadrilateral is 360° .

b) In a parallelogram diagonals bisect each other and opposite angles are

c) congruent.

d) A rhombus and a square have perpendicular diagonals. (Kite too)

e) A trapezoid is a quadrilateral with only one pair of parallel sides.

f) A square is a quadrilateral with four (all) congruent sides and four (all) right angles.

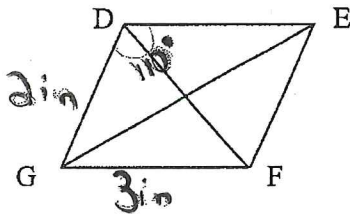
g) A rhombus is a parallelogram (or quadrilateral) with four congruent sides.

h) A parallelogram is a quadrilateral with 2 pairs of parallel sides.

i) Any four-sided polygon is a quadrilateral.

j) A rectangle is a quadrilateral with 4 right angles. (It also has 2 pairs of parallel sides, 2 pairs of congruent sides, and congruent, bisecting diagonals... but these are not part of the definition.)

5) Polygon DEFG is a parallelogram. $GF = 3$ in, $DG = 2$ in, $m\angle GDE = 110^\circ$



a) $m\angle DGF =$ 70°

b) $m\angle GFE =$ 110°

Should say EF (without the bar)

c) $\overline{EF} =$ 2 in

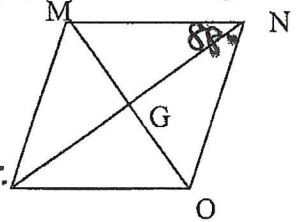
d) $\overline{DE} =$ 3 in
Should say DE (without the bar)

6) $MNOP$ is a rhombus. If $m\angle MNO = 88^\circ$, find each of the following:

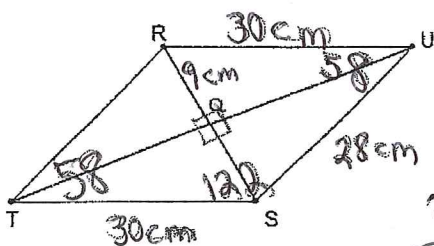
a) $m\angle NOP =$ 92°

b) $m\angle OPG =$ 44° (in a rhombus, diagonals bisect the opposite angles.)

c) $m\angle OGN =$ 90° (in a rhombus, diagonals are \perp)



7) Parallelogram $RUST$



$m\angle RUS =$ 58°

$m\angle UST =$ 122°

$m\angle STR =$ 58°

$m\angle TRU =$ 122°

$RU =$ 30 cm

$US =$ 28 cm

$ST =$ 30 cm

$TR =$ 28 cm

$RS =$ 18 cm

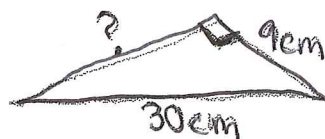
$RQ =$ 9 cm

$QS =$ 9 cm

$TQ =$ about 28.62 cm

$QU =$ about 28.62 cm

$UT =$ 30 cm



$9^2 + TQ^2 = 30^2$
 $TQ \approx 28.62$

57.24
4 cm

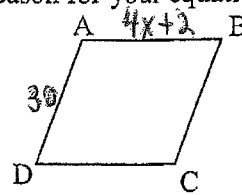
8) Polygon $ABCD$ is a rhombus. $AB = 4x + 2$ and $AD = 30$. What is x ? Give a reason for your equation.

$$4x + 2 = 30$$

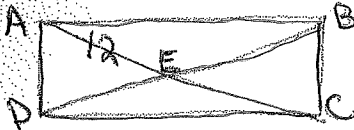
$$4x = 28$$

$$x = 7$$

All sides of a rhombus are congruent.



9) Polygon $ABCD$ is a rectangle. \overline{AC} and \overline{BD} intersect to E . $AE = 12$ ft. What is BD ?



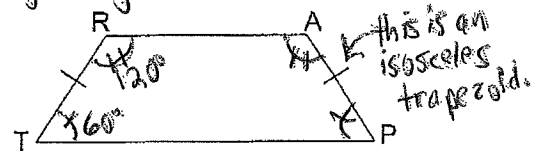
If $AE = 12$, then $AC = 24$.

If $AC = 24$, then $BD = 24$ because in a rectangle, diagonals are always congruent.

10) Use trapezoid $TRAP$ to the right to answer the following:

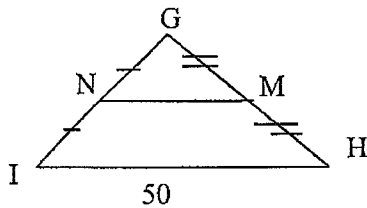
If $m\angle T = 60^\circ$ find the measures of the other angles.

$$m\angle R = 120^\circ \quad m\angle A = 120^\circ \quad m\angle P = 60^\circ$$

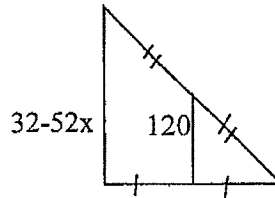


11) Find the following.

a) $NM = 25$



b) $x = -4$



$$2(120) = 32 - 52x$$

$$240 = 32 - 52x$$

$$208 = -52x$$

$$x = -4$$

c) What is \overline{NM} called? midsegment

12) Find the slope, midpoint, and length of each of the following segments whose endpoints are given.

a) $(-1, 4)$ and $(4, 10)$

slope: $\frac{4-10}{-1-4} = \frac{-6}{-5} = \frac{6}{5}$

midpoint: $(\frac{-1+4}{2}, \frac{4+10}{2}) = (\frac{3}{2}, 7)$

length: $\sqrt{(-1-4)^2 + (4-10)^2}$
 $= \sqrt{(-5)^2 + (-6)^2} = \sqrt{25+36} = \sqrt{61}$ or ≈ 7.81

b) $(8, 0)$ and $(10, 6)$

slope: $\frac{0-6}{8-10} = \frac{-6}{-2} = 3$

midpoint: $(\frac{8+10}{2}, \frac{0+6}{2}) = (9, 3)$

length: $\sqrt{(8-10)^2 + (0-6)^2}$
 $= \sqrt{(-2)^2 + (-6)^2} = \sqrt{4+36} = \sqrt{40} \approx 6.32$

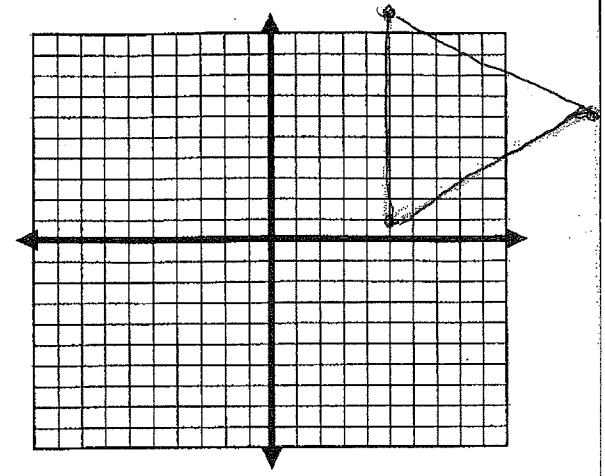
13) Lines that are parallel have same slopes and lines that are perpendicular have opposite reciprocal slopes.

14) Triangle TRI has vertices $T(15,6)$, $R(5,1)$, and $I(5,11)$. Use coordinate geometry to determine if triangle TRI is scalene, isosceles, or equilateral.

Do distance formula 3 times:

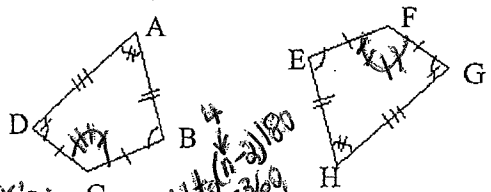
$$\begin{aligned} (15,6) &+ (5,1): & (5,1) &+ (5,11): & (15,6) &+ (5,11): \\ \sqrt{(15-5)^2 + (6-1)^2} & & \sqrt{(5-5)^2 + (1-11)^2} & & \sqrt{(15-5)^2 + (6-11)^2} \\ = \sqrt{(10)^2 + (5)^2} & & = \sqrt{(0)^2 + (-10)^2} & & = \sqrt{(10)^2 + (-5)^2} \\ = \sqrt{100+25} & & = \sqrt{100} & & = \sqrt{100+25} \\ = \sqrt{125} & & = 10 & & = \sqrt{125} \end{aligned}$$

2 match so it's isosceles



Section 4

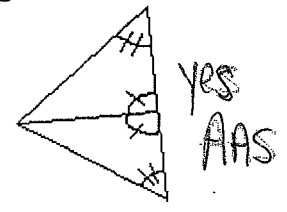
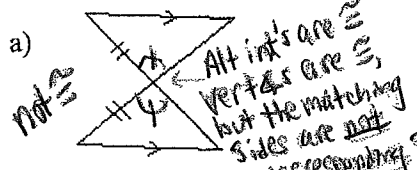
1) Write a congruency statement for the following polygons. Why are they congruent?



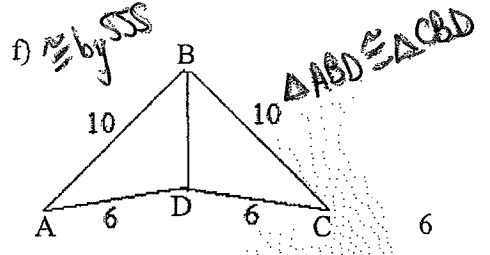
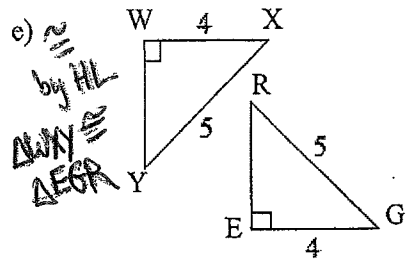
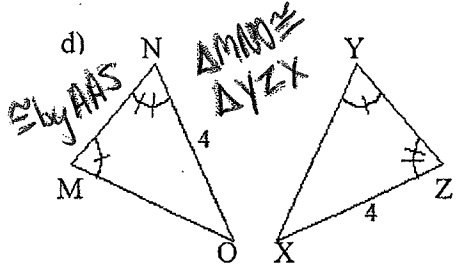
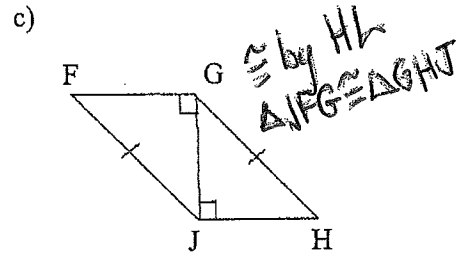
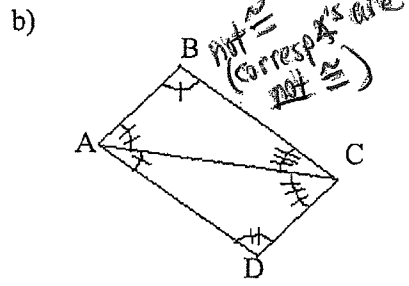
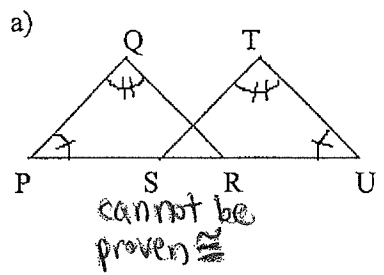
$ABCD \cong HEFG$
All sides are \cong
and
All corr. \angle 's are \cong

(since all \angle 's have to add to $(n-2)180$
then $m\angle C$ will = $m\angle D$ automatically.)

2) Determine whether each pair of triangles can be proven congruent by using the SSS, SAS, ASA or AAS congruence postulates. If so, identify what postulate is used.

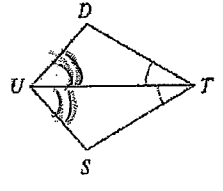


3) Determine whether each pair of triangle scan be proven congruent by using the SSS, SAS, ASA, AAS or HL congruence postulates. If so, identify what postulate is used and write a congruency statement.

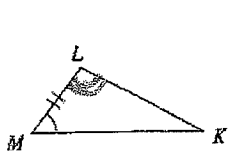


4) Label and state what additional information is required in order to know that the triangles are congruent for the reason given.

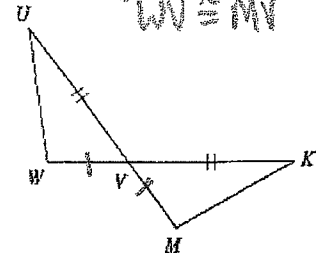
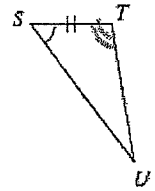
a) ASA *you'd need $\angle DUT \cong \angle SUT$*



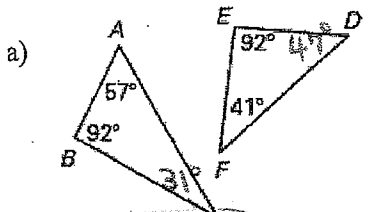
b) ASA *you'd need $\angle L \cong \angle T$*



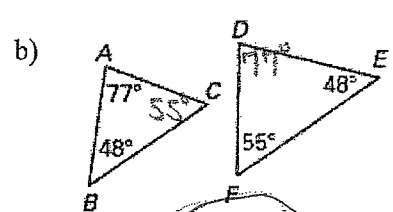
c) SAS *you'd need $\overline{UV} \cong \overline{MV}$*



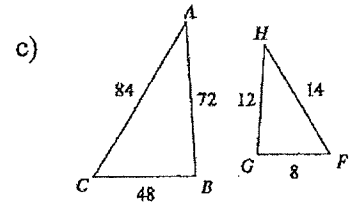
5) Determine whether or not the triangles below are similar (you may need to do a little work to figure it out) by AA, SSS, or SAS, or none of them. If they are similar, complete the similarity statement.



$\triangle ABC$ *Not similar*

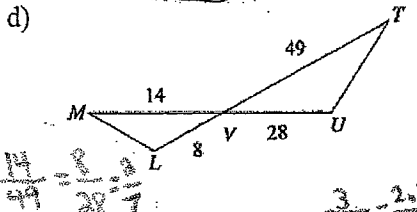


$\triangle ABC \sim \triangle DEF$

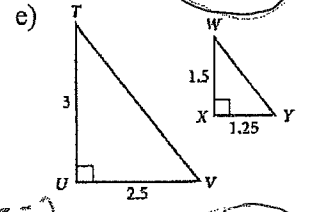


$$\frac{48}{8} = \frac{72}{12} = \frac{84}{14} = 6$$

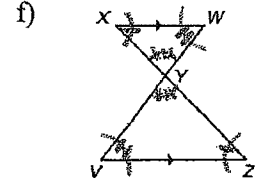
$\triangle CBA \sim \triangle FGH$



$\triangle LVM \sim \triangle UVT$

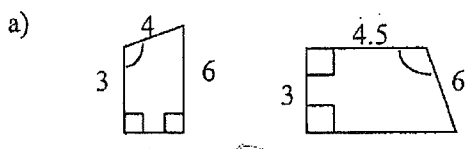


$\triangle TUV \sim \triangle WXY$



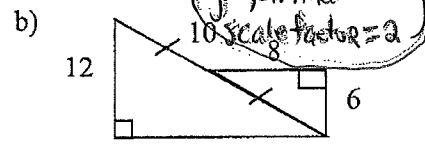
$\triangle WXY \sim \triangle VZY$

6) Determine whether the polygons are similar, not similar, or not enough information given. If they are similar, determine the scale factor comparing the first to second figure.



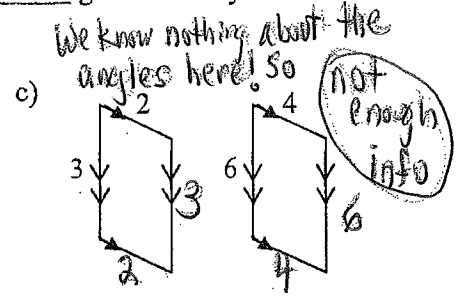
$$\frac{3}{4.5} = \frac{4}{6} = \frac{6}{9} = \frac{2}{3} = \frac{1}{1.5}$$

Yes, similar scale factor = 2:3 or .6



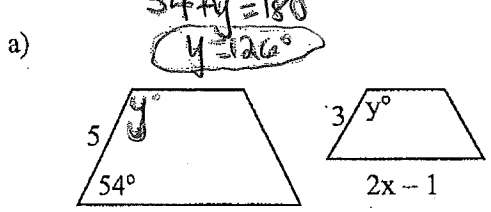
$$\frac{6}{12} = \frac{16}{20} = \frac{10}{20} = \frac{1}{2}$$

yes, similar 10 scale factor = 2



We know nothing about the angles here! So not enough info

7) The following polygons are similar, find x and y.

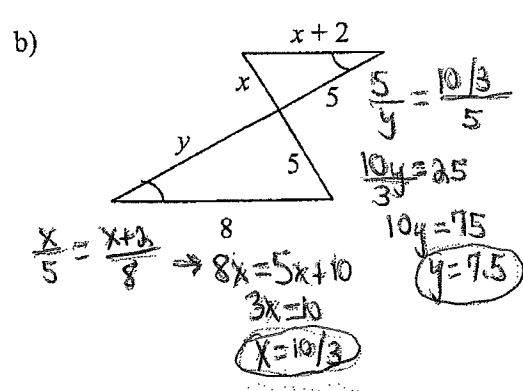


$$\frac{5}{1} \times \frac{3}{2x-1} \rightarrow \frac{1}{1} \times \frac{3}{2x-1} = \frac{5}{54}$$

$$10x - 5 = 3$$

$$10x = 8$$

$$x = .8$$



$$\frac{5}{5} = \frac{10/3}{5}$$

$$\frac{10y}{3} = 25$$

$$10y = 75$$

$$y = 7.5$$

$$\frac{x}{5} = \frac{x+2}{8} \rightarrow 8x = 5x + 10$$

$$3x = 10$$

$$x = 10/3$$

8) $\triangle AFN \sim \triangle DPG$, $AF = 2$ cm., $FN = 3$ cm., $DG = 10$ cm., and $PD = 8$ cm. Find AN . If $m\angle A = 36^\circ$, what is



$$\frac{2}{x} = \frac{8}{10}$$

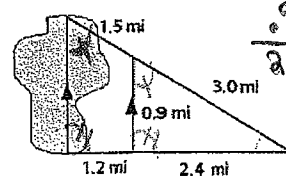
$$8x = 20$$

$$x = 2.5$$

$m\angle D = 36^\circ$ also

9) Use the following image to explain why the two triangles are similar, then estimate the length of the lake.

They're similar because the segments are parallel, creating congruent corresponding angles. Also both \triangle 's share the third angle, so that angle is congruent to itself. However the shortcut AA only requires 2 pairs of angles to be congruent.

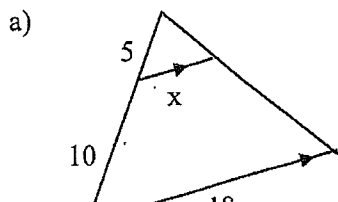


$$\frac{0.9}{2.4} = \frac{x}{3.0}$$

$$2.4x = 3.24$$

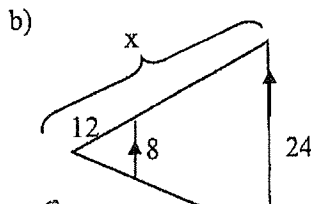
$$x = 1.35 \text{ miles}$$

10) Solve for x.



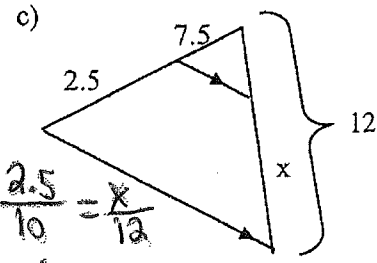
$$\frac{5}{x} = \frac{15}{18} \rightarrow 15x = 90$$

$$x = 6$$



$$\frac{8}{24} = \frac{12}{x} \rightarrow 8x = 288$$

$$x = 36$$

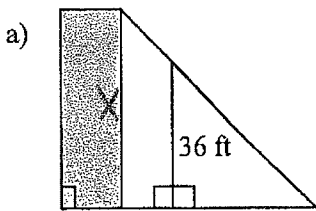


$$\frac{2.5}{10} = \frac{x}{12}$$

$$10x = 30$$

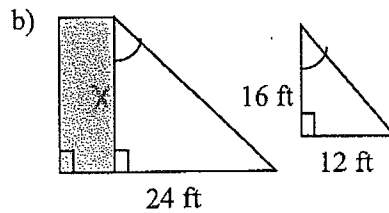
$$x = 3$$

11) Use the diagram to find the height of each building.



$$\frac{40}{70} = \frac{36}{x} \rightarrow 40x = 2520$$

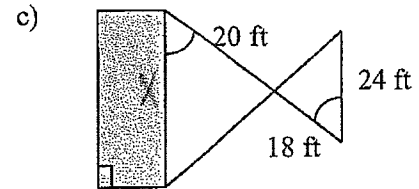
$$x = 63 \text{ ft}$$



$$\frac{x}{16} = \frac{24}{12}$$

$$12x = 384$$

$$x = 32 \text{ ft}$$



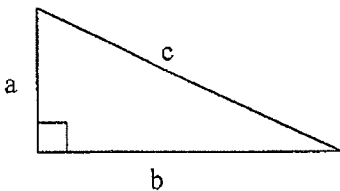
$$\frac{x}{24} = \frac{20}{18}$$

$$18x = 480$$

$$x = 26.6 \text{ ft}$$

Section 5

1) For # 1-3 two lengths of the right triangle are given. Find the missing length.



a) $a = 13$
 $13^2 + b^2 = 14^2$
 $169 + b^2 = 196 \rightarrow b^2 = 27 \rightarrow b \approx 5.20$

$b = \approx 5.20$ or $\sqrt{27}$ $c = 14$

b) $a = 12$
 $12^2 + 16^2 = c^2$

$b = 16$ $c = 20$

c) $a = \approx 10.95$ or $\sqrt{120}$
 $a^2 + 17^2 = 13^2$

$b = 7$ $c = 13$

2) A triangle has side lengths given below. Determine what type of triangle each set is (acute, obtuse, or right). Show work to support your answer.

a. 24, 40, and 32

$$40^2 = 24^2 + 32^2$$

$$1600 = 576 + 1024$$

$$1600 = 1600$$

right

b. 30, 24, and 19

$$30^2 = 24^2 + 19^2$$

$$900 = 576 + 361$$

$$900 < 937 \text{ (acute)}$$

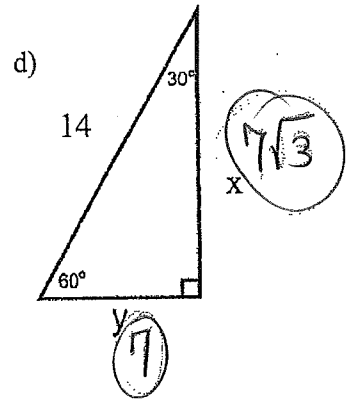
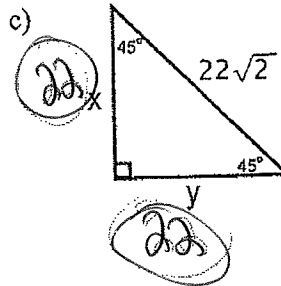
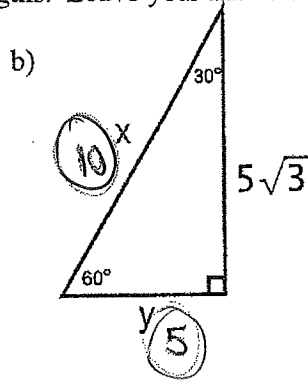
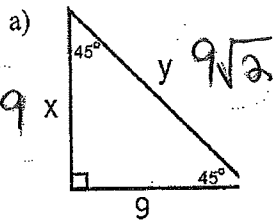
c. 6, 14, and 11

$$14^2 = 6^2 + 11^2$$

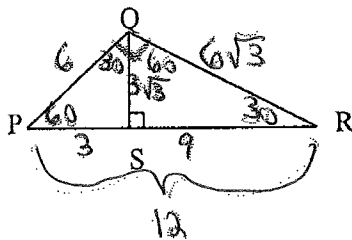
$$196 = 36 + 121$$

$$196 > 157 \text{ (obtuse)}$$

3) Find the missing side lengths. Leave your answers in radical form.



4) For the following, ΔPQR , $m\angle PQR = 90^\circ$, $PQ = 6$, $m\angle QPS = 60^\circ$, and $PR = 12$.



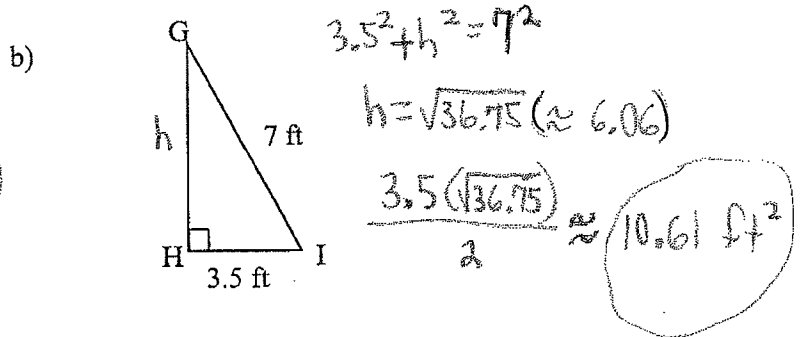
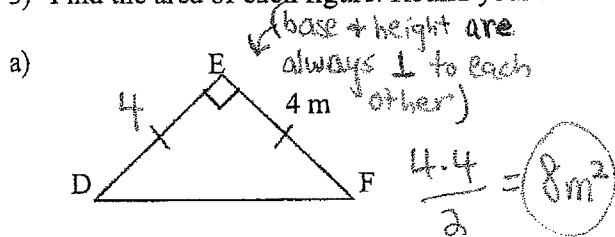
a) Find $QR = 6\sqrt{3}$ or ≈ 10.39

b) Find $QS = 3\sqrt{3}$ or ≈ 5.20

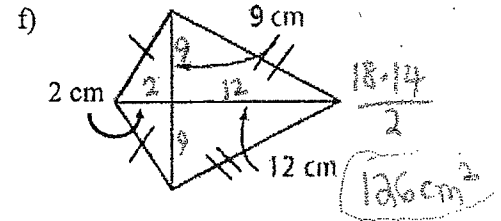
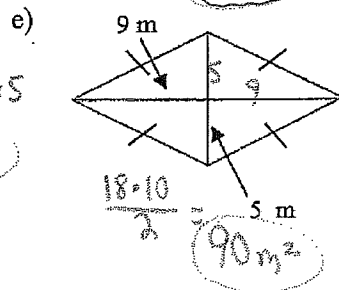
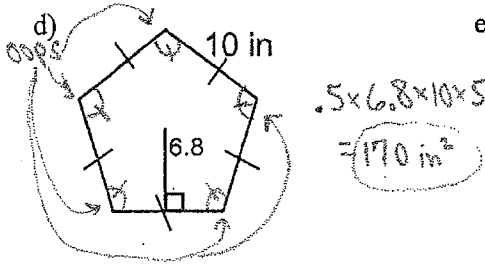
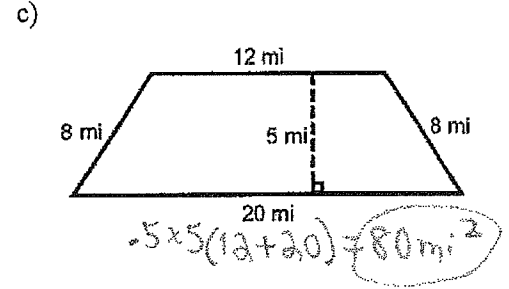
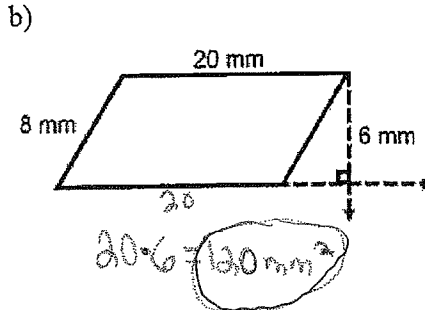
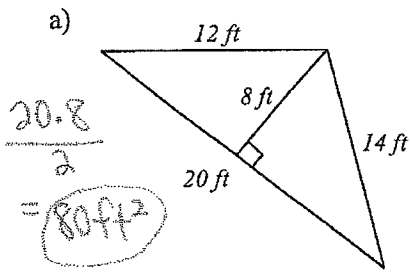
c) Find $SR = 9$

d) Find the area of $\Delta PQR = \frac{bh}{2} = \frac{12 \cdot 3\sqrt{3}}{2} = 18\sqrt{3} \text{ u}^2$ or $\approx 31.18 \text{ u}^2$

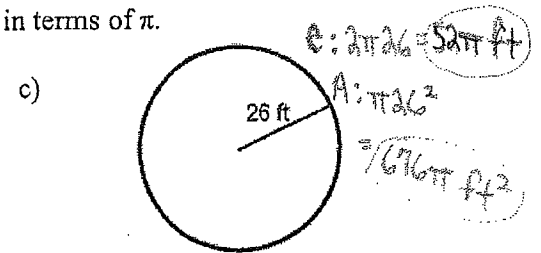
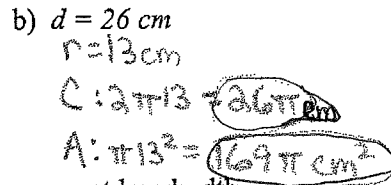
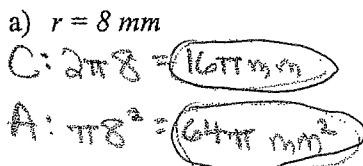
5) Find the area of each figure. Round your answers to the nearest tenth.



6) Find the area of the following figures.



7) Find the circumference AND area of each figure. Leave your answer in terms of π .



8) Round your answers to ~~15~~ to the nearest hundredth.

$C = \underline{50.27 \text{ mm}}$ $A = \underline{201.06 \text{ mm}^2}$

9) Find the radius of each circle from the given information. Round to the nearest tenth if necessary.

a) Area = $256\pi \text{ in}^2$
 $\pi r^2 = 256\pi$
 $r^2 = 256$
 $r = 16 \text{ in}$

b) Circumference = 120 ft
 $\frac{120}{2\pi} = \frac{2\pi r}{2\pi}$
 $r = 19.1 \text{ ft}$

10) If the area of a parallelogram is 100 cm^2 and the length of the base is 25 cm , what is the height?

$A = bh \rightarrow 25h = 100$
 $h = 4 \text{ cm}$

11) If the area of a parallelogram is 45 ft^2 and the height is 3 ft , what is the length of the base?

$A = bh \rightarrow 45 = b(3)$
 $b = 15 \text{ ft}$

12) If the area of a trapezoid is 250 in^2 , the lengths of the bases are 23 in and 27 in , what is the height?

$A = .5h(b_1 + b_2) \rightarrow 250 = .5h(23 + 27)$
 $250 = .5h(50)$
 $250 = 25h$
 $h = 10 \text{ in}$

13) If the area of a triangle is $343 u^2$ and the height is $14 u$, what is the length of the base?

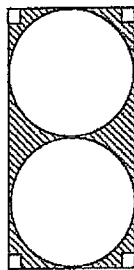
$$A = .5bh \rightarrow 343 = .5b(14)$$

$$343 = 7b \quad b = 49 u$$

14) Find the area of the shaded region.

$$(20 \times 40) - 2(\pi 10^2)$$

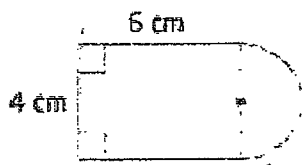
$$800 - 200\pi \approx 171.68 u^2$$



20

15) Find the area of the composite figures below.

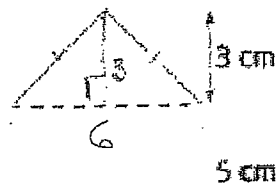
a)



$$24 + .5(\pi 2^2)$$

$$24 + 2\pi \approx 30.28 \text{ cm}^2$$

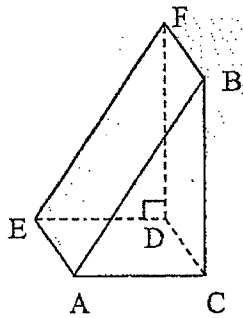
b)



$$30 + .5(6)(3) = 39 \text{ cm}^2$$

Section 6

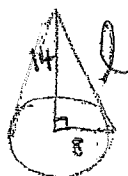
1) For the following, refer to the solid below.



- Name the solid. triangular prism
- Name a pair of parallel planes. $\triangle ABC$ and $\triangle EDF$
- Name two segments skew to \overline{BF} . \overline{AC} and \overline{ED}
- Name two segments \perp to plane BFD. \overline{AC} and \overline{ED}
- What is the volume of the solid if $BC = 4$, $AC = 3$, and $DC = 2$.

$$V = (.5 \times 3 \times 4) \times 2 = 12 u^3$$

2) What is the slant height of a right cone with a radius of 8 in. and a height of 14 in. $\sqrt{260}$ or ≈ 16.12 inches



$$8^2 + 14^2 = 260$$

Find the Surface Area and Volume of each right prism. Round to the hundredth if necessary.

3) Find the Surface Area, Lateral Area, and Volume for the following solids. Give an exact answer.

a. $A = 2lw + 2wh + 2lh$
 $= 72 \text{ cm}^2$
 $V = lwh = 32 \text{ cm}^3$

lateral area always is everything but the base areas, so $2lw + 2lh$ only.
 $2(8 \cdot 2) = 64 \text{ cm}^2$

b. $SA = hp + 2B$
 $LA = hp$
 $V = Bh$
 $V = 18 \text{ in}^3$
 $SA = 3(3+4+5) + 2(5 \cdot 3 \cdot 4) = 48 \text{ in}^2$
 $LA = 36 \text{ in}^2$

c. $BC = 12, CH = 5, \text{apothem} = 6.2$
 $SA = 5(12 \cdot 5) + 2(5 \cdot 6.2 \cdot 12)$
 $= 672 \text{ in}^2$
 $LA = 300 \text{ in}^2$
 $V = 930 \text{ in}^3$

d. $SA = 2\pi r^2 + 2\pi rh$
 $LA = 2\pi rh$
 $V = \pi r^2 h$
 $SA = 376.99 \text{ cm}^2$
 $LA = 1207 \text{ cm}^2$
 $V = 452.39 \text{ cm}^3$
 $144\pi \text{ cm}^2$
 $78\pi \text{ cm}^2$

e. $SA = 2\pi r^2 + 2\pi rh$
 $LA = 2\pi rh$
 $V = \pi r^2 h$
 $SA = 282.74 \text{ ft}^2$
 $LA = 90\pi \text{ ft}^2$
 $V = 914.16 \text{ ft}^3$
 $100\pi \text{ ft}^2$

f. $SA = 5lp + B$
 $LA = 5lp$
 $V = \frac{Bh}{3}$
 $SA = 202.4 \text{ in}^2$
 $V = 256 \text{ in}^3$

g. $3^2 + h^2 = 9^2$
 $h = 8.49$
 $SA = 144 \text{ cm}^2$
 $LA = 108 \text{ cm}^2$
 $V = 101.98 \text{ cm}^3$

(Let's say $l = 12$ Even though it's not very clear here)

h. $\text{apothem} = 5.2 \text{ cm}$
 $SA = 5(12 \cdot 6 \cdot 6) + 5(5.2 \cdot 6 \cdot 6)$
 $SA = 309.6 \text{ cm}^2$
 $LA = 5(12 \cdot 6 \cdot 6) = 216 \text{ cm}^2$
 $V = \frac{5 \cdot 5.2 \cdot 6 \cdot 6}{3} = 362.54 \text{ cm}^3$

i. $SA = \pi r l + \pi r^2$
 $LA = \pi r l$
 $V = \frac{\pi r^2 h}{3}$
 $SA = 785.40 \text{ cm}^2 = 250\pi \text{ cm}^2$
 $LA = 471.24 \text{ cm}^2 = 150\pi \text{ cm}^2$
 $V = 117.08 \text{ cm}^3 = 37.26\pi \text{ cm}^3$

j. $12^2 + 16^2 = l^2$
 $l = 20$
 $SA = 1306.37 \text{ in}^2 = 384\pi \text{ in}^2$
 $V = 242.74 \text{ in}^3 = 768\pi \text{ in}^3$
 $LA = 753.98 \text{ in}^2 = 240\pi \text{ in}^2$

k. $SA = 4\pi r^2$
 $V = \frac{4\pi r^3}{3}$
 $SA = 452.39 \text{ cm}^2 = 144\pi \text{ cm}^2$
 $V = 904.78 \text{ cm}^3 = 288\pi \text{ cm}^3$

l. $4\pi r^2$
 $= 1017.88 \text{ in}^2 = 324\pi \text{ in}^2$
 $V = 6053.63 \text{ in}^3 = 972\pi \text{ in}^3$

m. $SA = 2lw + 2wh + 2lh + 5lp$
 $= 288 \text{ cm}^2$
 $V = lwh + \frac{Bh}{3}$
 $= 336 \text{ cm}^3$

don't include shaded region in SA

n. $SA = \pi r l + 5(4\pi r^2)$
 $= \pi \cdot 2 \cdot 10.20 + 5(4\pi \cdot 2^2)$
 $= 89.22 \text{ in}^2 = 28.4\pi \text{ in}^2$
 $V = \frac{\pi r^2 h}{3} + 5(\frac{4\pi r^3}{3})$
 $= \frac{\pi \cdot 2^2 \cdot 10}{3} + 5(\frac{4\pi \cdot 2^3}{3})$
 $= 58.64 \text{ in}^3 = 18.6\pi \text{ in}^3$

don't include in SA

o. $SA = 2\pi rh + 1\pi r^2 + \pi r l$
 $= 203.58 \text{ cm}^2 = 64.8\pi \text{ cm}^2$
 $V = \pi r^2 h + \frac{\pi r^2 h}{3}$
 $= \pi \cdot 3^2 \cdot 5.1 + \frac{\pi \cdot 3^2 \cdot 7.85}{3}$
 $= 218.18 \text{ cm}^3 = 69.45\pi \text{ cm}^3$

don't include h of cone $\rightarrow 3^2 + h^2 = 8.4^2 \rightarrow h = 7.85$

4) The surface area of a square pyramid is given by 540 cm^2 and the side of the square is 10 cm . Find the slant height of the square pyramid. $SA = .5lp + B \rightarrow 540 = .5l(10 \times 4) + 10^2$

$540 = 20l + 100$
 $440 = 20l$
 $l = 22 \text{ cm}$

5) The volume of a cylinder is 960π cubic inches. The height of the cylinder is 15 inches. Find the radius.

$V = \pi r^2 h \rightarrow 960\pi = \pi r^2 15 \rightarrow \frac{960}{15} = \frac{15r^2}{15} \rightarrow r^2 = 64 \rightarrow r = 8 \text{ in}$

6) If a cylinder has surface area of 128π sq ft, and the height of the cylinder is 12 feet, find the radius and the volume.

$128\pi = 2\pi r^2 + 2\pi r(12)$
 $128 = 2r^2 + 2r(12)$
 $128 = 2r^2 + 24r$
 $64 = r^2 + 12r$
 $r^2 + 12r - 64 = 0$
 $(r-4)(r+16) = 0$
 $r = 4, r = -16$

$V = \pi r^2 h$
 $V = \pi(4^2)(12)$
 $V = 192\pi \text{ ft}^3 \approx 603.19 \text{ ft}^3$

7) The volume of a spherical ball is $5,000\pi \text{ cm}^3$. What is the radius of the ball?

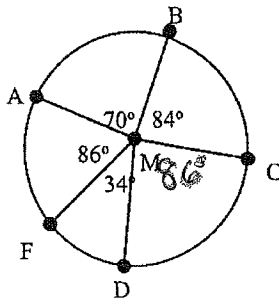
$V = \frac{4\pi r^3}{3} \rightarrow 5000\pi = \frac{4\pi r^3}{3} \rightarrow 5000 = \frac{4r^3}{3}$

$\rightarrow \frac{15000}{4} = \frac{4r^3}{4}$

$r^3 = 3750 \rightarrow r \approx 15.54 \text{ cm}$

Section 7

1) Find the degree measures of each arc or angle by using the central angle measures given in $\odot M$



a) $m\widehat{AC} = 154^\circ$

b) $m\widehat{FA} = 86^\circ$

c) $m\widehat{CBF} = 240^\circ$

d) $m\widehat{DB} = 170^\circ$

e) $m\widehat{ADC} = 206^\circ$

f) $m\widehat{DCA} = 240^\circ$

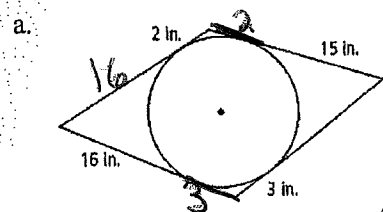
g) $m\angle DMC = 86^\circ$

2) Determine arc with length L of a circle with radius 8.5 in and degree measure 240° .

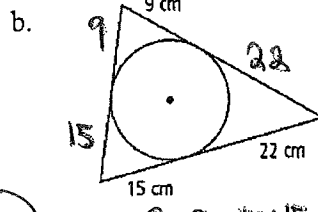
$\frac{m}{360} = \frac{L}{2\pi r} \rightarrow \frac{240}{360} = \frac{L}{2\pi 8.5} \rightarrow 360L = 240 \times 2\pi 8.5$

$\frac{360L}{360} = \frac{4080\pi}{360} \rightarrow L = 11.3\pi \text{ in} \approx 35.60 \text{ in}$

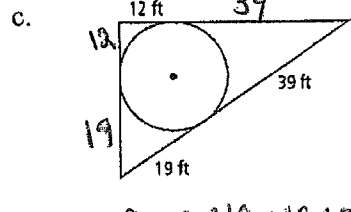
3) Each polygon circumscribes a circle. What is the perimeter of each polygon?



$16 + 16 + 2 + 2 + 15 + 15 + 3 + 3 = 72 \text{ in}$



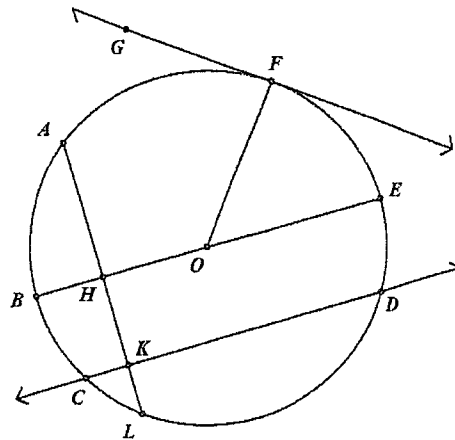
$9 + 9 + 15 + 15 + 22 + 22 = 92 \text{ cm}$



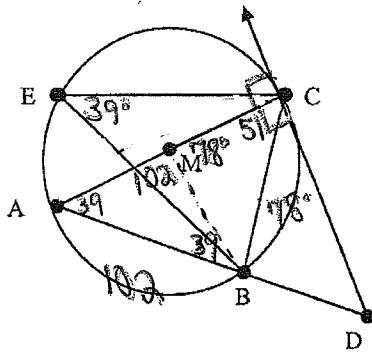
$12 + 12 + 19 + 19 + 39 + 39 = 140 \text{ ft}$

4) Using circle O below, name the following:

- a. Diameter \overline{OE}
- b. Central Angle sample answer: $\angle FDE$
- c. Minor Arc sample answer: \widehat{AB}
- d. Major Arc sample answer: \widehat{FLA}
- e. Semicircle sample answer: \widehat{BDE}
- f. Radius sample answer: \overline{OF}
- g. Tangent \overline{GF}
- h. Point of Tangency Point F



5) For the following, in $\odot M$, \overline{AC} is the diameter, \overline{DC} is tangent to the circle at point C , and $m\widehat{BC} = 78^\circ$.



- a) $m\angle BAC$ 39°
- b) $m\angle BEC$ 39°
- c) $m\widehat{AB}$ 102°
- d) $m\angle ACB$ 51°
- e) $m\angle ABC$ 90°
- f) $m\angle ACD$ 90°

Sample Answers:

- g) \widehat{AB} is a minor arc, \widehat{CEB} is a major arc
- h) \overline{MA} is a radius, \overline{AC} is a diameter
- i) \overline{CD} is a tangent line

Find the measure of the arc or angle in $\odot M$.

7. $m\angle QMP$ 60°

8. $m\angle NMO$ 110°

9. $m\angle PNO$ 35°

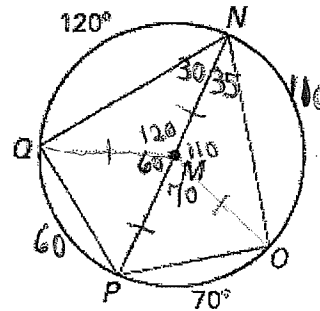
10. $m\angle QNP$ 30°

11. $m\widehat{QO}$ 130°

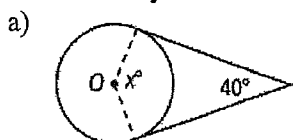
12. $m\widehat{NOP}$ 180°

13. $m\widehat{PQ}$ 60°

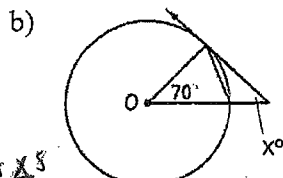
14. $m\widehat{QON}$ 250°



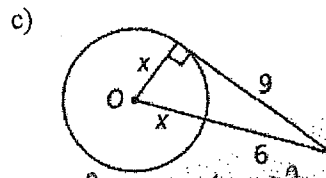
15) What is the value of x ? Lines that appear to be tangent are tangent. Round to the nearest hundredth if necessary.



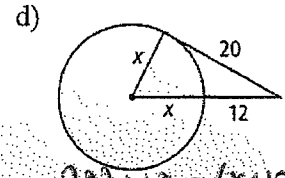
$(n-2)180 = \text{sum of interior angles}$
 $(4-2)180 = 360$
 $90, x + 90 + 90 + 40 = 360$
 $x = 140^\circ$



$90 + 70 + x = 180$
 $x = 20^\circ$



$x^2 + 9^2 = (x+6)^2$
 $x^2 + 81 = x^2 + 12x + 36$
 $81 = 12x + 36$
 $45 = 12x$
 $x = 3.75$



$20^2 + x^2 = (x+12)^2$
 $x^2 + 400 = x^2 + 24x + 144$
 $400 = 24x + 144$
 $256 = 24x$
 $x = 10.6$

16) Write the equation for the circle with center (2, 4) and radius = 7 in

$$(x-2)^2 + (y-4)^2 = 49$$

17) Write the equation for the circle with center (-3, 1) and diameter = 18 in

$$(x+3)^2 + (y-1)^2 = 81 \quad (\text{remember, if } d=18, \text{ then } r=9)$$

18) Find the center and radius of the circle: $(x-7)^2 + (y+12)^2 = 144$

$$\text{Center: } (7, -12), \text{ radius: } 12$$

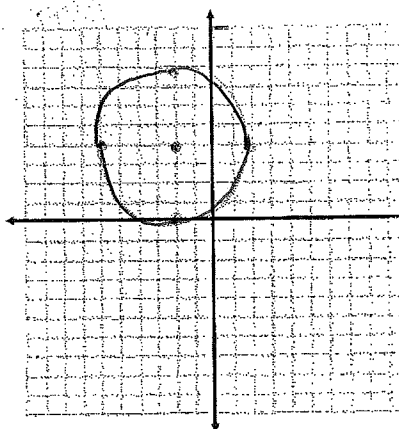
19) Find the center and radius of the circle: $(x+5)^2 + (y+8)^2 = 225$

$$\text{Center: } (-5, -8)$$

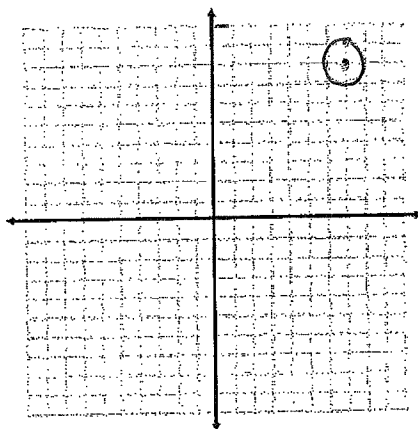
$$\text{radius: } 15$$

20) Graph the circle on the coordinate plane.

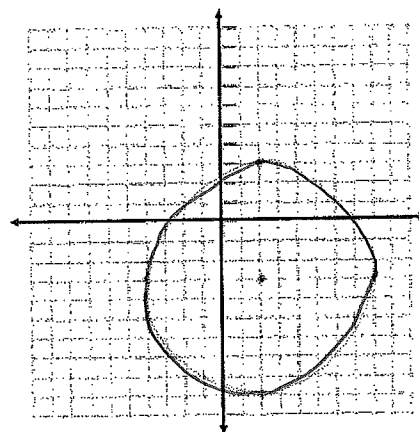
a. $(x+2)^2 + (y-4)^2 = 16$



b. $(x-7)^2 + (y-8)^2 = 1$



c. $(x-2)^2 + (y+3)^2 = 36$



Find the center and radius of the following equations by completing the square:

$$y^2 + 4x - 20 - 2y = -x^2$$

$$x^2 + 4x + y^2 - 2y - 20 = 0$$

$$x^2 + 4x + 4 + y^2 - 2y + 1 = 20 + 4 + 1$$

$$(x+2)^2 + (y-1)^2 = 25$$

$$\text{Center: } (-2, 1)$$

$$\text{radius: } 5$$

$$16 + x^2 + y^2 - 8x - 6y = 0$$

$$x^2 - 8x + y^2 - 6y = -16$$

$$x^2 - 8x + 16 + y^2 - 6y + 9 = -16 + 16 + 9$$

$$(x-4)^2 + (y-3)^2 = 9$$

$$\text{Center: } (4, 3)$$

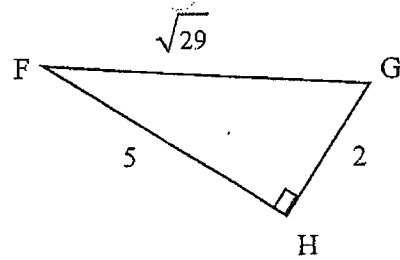
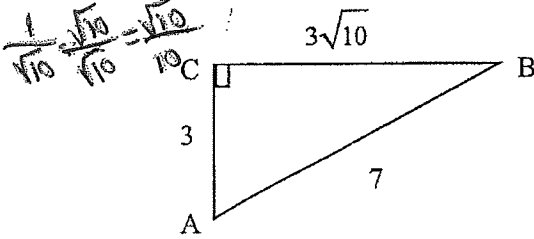
$$\text{radius: } 3$$

Section 8

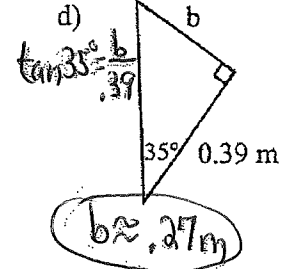
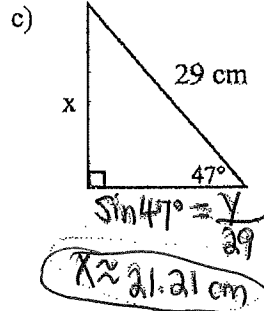
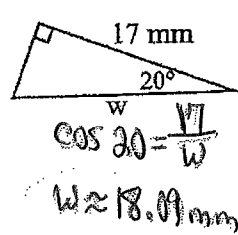
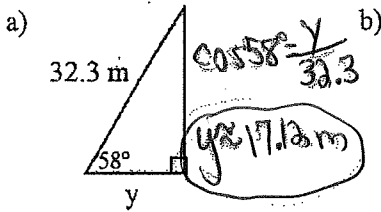
1) Using the triangles below, determine the trigonometric ratio. Leave your answers as simplified fractions.

a) $\tan B = \frac{3}{3\sqrt{10}} = \frac{1}{\sqrt{10}}$ $\cos A = \frac{3}{7}$

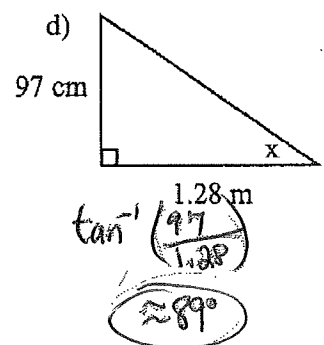
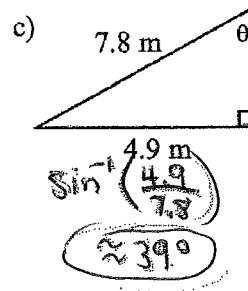
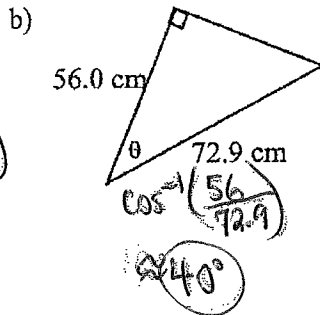
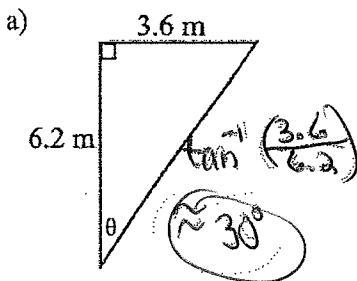
c) $\sin F = \frac{2}{\sqrt{29}}$ d) $\tan G = \frac{5}{2}$



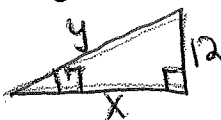
2) Find the marked side of each of the following triangles.



3) Find the value for each of the marked angles.



4) A skateboarding ramp is 12 in. high and rises at an angle of 17°. How long is the base of the ramp? What is the length of the ramp? Round your answer to the nearest inch.



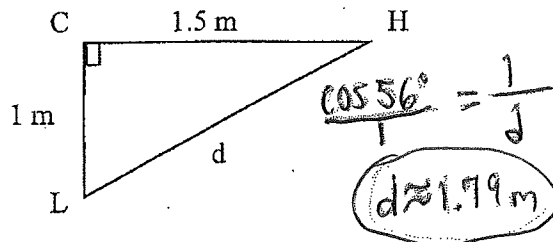
$\tan 17^\circ = \frac{12}{x}$
 $x \approx 39 \text{ in}$

$\sin 17^\circ = \frac{12}{y}$
 $y \approx 41 \text{ in}$

5) Joey is walking home from the library. He can walk for 1 mile along the street, then turn right and walk 1.5 miles along another street; or he can cut across a large field straight to his house. At what angle, θ , should he head off from the library, and how far, d , should he cut across the field?

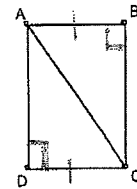
$\theta = \text{about } 56^\circ = \tan^{-1}(\frac{1.5}{1})$

$d = \text{about } 1.79 \text{ m}$



Proofs

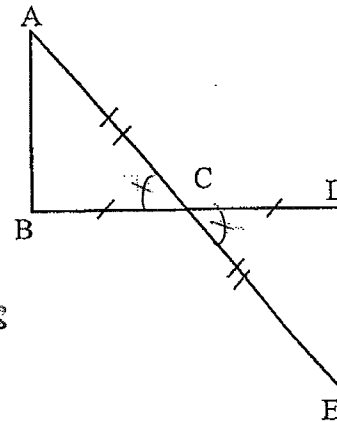
1) Given: $\angle B$ and $\angle D$ are right angles, $\overline{AB} \cong \overline{CD}$
 Prove: $\angle DAC \cong \angle BCA$



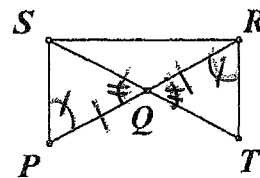
Statements	Reasons
① $\angle B$ & $\angle D$ are right angles; $\overline{AB} \cong \overline{CD}$	① Given
② $\angle B \cong \angle D$	② All right angles are congruent
③ $\overline{AC} \cong \overline{CA}$	③ Reflexive Property of congruence
④ $\triangle ABC \cong \triangle CDA$	④ HL
⑤ $\angle DAC \cong \angle BCA$	⑤ CPCTC

Given: $\overline{AC} \cong \overline{EC}$, $\overline{BC} \cong \overline{DC}$
 Prove: $\angle CBA \cong \angle CDE$

Statements	Reasons
① $\overline{AC} \cong \overline{EC}$; $\overline{BC} \cong \overline{DC}$	① Given
② $\angle ACB$ and $\angle ECD$ are vertical angles	② Definition of Vertical Angles
③ $\angle ACB \cong \angle ECD$	③ Vertical Angles Theorem
④ $\triangle CBA \cong \triangle CDE$	④ SAS
⑤ $\angle CBA \cong \angle CDE$	⑤ CPCTC



Given: Q is the midpoint of \overline{PR} , $\angle P \cong \angle R$
 Prove: $\triangle SQP \cong \triangle TQR$



Statements	Reasons
① Q is the midpoint of \overline{PR} ; $\angle P \cong \angle R$	① Given
② $\overline{PQ} \cong \overline{RQ}$	② Definition of midpoint
③ $\angle SQP$ & $\angle TQR$ are vertical angles	③ Definition of Vertical Angles
④ $\angle SQP \cong \angle TQR$	④ Vertical Angles Theorem
⑤ $\triangle SQP \cong \triangle TQR$	⑤ ASA